Experimental and theoretical aspects of quantum chaos

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Abstract

In this series of lectures an introduction into quantum chaos is presented. The discussion will be kept on an elementary level, and theory will be illustrated, wherever possible, by experimental or numerical examples. Microwave billiards will play a major role in this respect.

The lectures start with a presentation of the various types of billiard experiments. Mesoscopic systems are discussed as well, as far as they are of the billiard type. In the second part the essential ideas of random matrix theory are presented, including an introduction into supersymmetry. Spectral level dynamics, when an external parameter is varied, shows a close analogy to the dynamics of a one-dimensional gas with repulsive interaction. This may be the explanation for success of random matrix theory in the interpretation of the universal spectral features of chaotic systems. The coupling of the billiards to the outer world is described in terms of scattering theory originally developed in nuclear physics. The course ends with an introduction into semiclassical physics, in particular the work of Gutzwiller and his trace formula establishing a connection between the spectrum of a system and its periodic orbits.

Lecture 1

Billiard experiments

Until about 1990 only a very small number of experiments on the quantum mechanics of chaotic systems existed, apart from the early studies of nuclear spectra [1]. This changed with the studies of irregularly shaped microwave cavities by Stöckmann and Stein [2]. The microwave billiards, vibrating blocks, and a number of variants to be discussed in this lecture are classical wave systems, which use the equivalence of the Helmholtz equation and the time independent Schrödinger equation [3].

Starting with a historical review, the state of the arts in billiard experiments is presented with emphasis on a general survey and the technical background. The results and their quantum mechanical implications will be presented later. Mesoscopic systems are discussed as well, but are restricted to billiard-like structures such as quantum dots, tunnelling devices, and quantum corrals.

Lecture 2

Random matrices

Random matrix theory has been developed already in the fifties and sixties of 20th century by Wigner, Dyson, Mehta and others [4, 5]. Originally conceived to bring some order into the spectra of complex nuclei, the interest in random matrix theory renewed when it was observed that random matrix theory seems to be able to describe the universal properties of the spectra of *all* chaotic systems.

In this lecture the basic concepts will be introduced, knowledge of which is indispensable already for beginners. Mathematical derivations will be presented only exemplarily to give an idea of the techniques applied. We shall concentrate on topics where also experimental material is available such as the nearest neighbour spacing distribution, number variance, spectral rigidity, and spectral form factor.

In the last part the basic ideas of the supersymmetry technique will be explained. It has become meanwhile the method of choice to treat ensemble averages [6].

Lecture 3

Spectral level dynamics

In many spectroscopic experiments the energy levels of a system are determined as a function of an external parameter. In mesoscopic systems usually an external magnetic field takes this role, in billiard systems it is a shape parameter such as length. In chaotic systems degeneracies do not occur, apart from accidental ones of measure zero. For two-level systems this effect is well-known from elementary quantum mechanics. The resulting motion of the eigenvalues in dependence of the external parameter strongly resembles the dynamics of the particles of a one-dimensional gas with a repulsive interaction.

The dynamical concept has been introduced by Pechukas [7] and further developed by Yukawa [8]. They showed that ordinary statistical mechanics can be applied to describe the eigenvalue dynamics of the spectra of chaotic systems. Random matrix theory then results as a direct consequence of the Boltzmann ansatz of statistical mechanics.

If the Hamiltonian depends on two parameters, closed loops are possible in the two-dimensional parameter space with the surprising consequence that the phases of the eigenfunctions may have changed, known as *Berry's* phases [9].

Lecture 4

Scattering systems

It is impossible to study a system without disturbing it by the measuring process. To determine, e. g., the spectrum of a microwave billiard, we have to introduce an antenna to irradiate the microwave field. The measurement thus unavoidably yields an unwanted combination of the system's own properties and those of the measuring apparatus. The mathematical tool to treat the coupling between the system and its environment is provided by scattering theory, which has originally been developed in nuclear physics [10]. Later this theory has been successfully applied to mesoscopic systems, as well as to microwave billiards.

In this lecture scattering theory will be introduced with special emphasis on billiard systems. Random matrix theory and the model of random superposition of plane waves will play an important role in this context. Mesoscopic systems can be linked to scattering theory via the *Landauer formula* [11] expressing the conduction through mesoscopic devices in terms of transmission probabilities.

Lecture 5

Semiclassical quantum mechanics

In the preceding lectures we learnt that random matrix theory is perfectly able to explain the *universal* properties of the spectra of chaotic systems, and this in spite of the oversimplifying assumptions applied. In this lecture we shall concentrate on the *indivudual* system properties. We know from the correspondence principle that in the semiclassical limit quantum mechanics must turn into classical mechanics. This was the starting point for Gutzwiller to develop his semiclassical quantum mechanics [12], culminating in the famous trace formula establishing a connection between the spectrum and the classical periodic orbits of a system.

In the present lecture the background of semiclassical quantum mechanics is sketched. The technique is applied to extract the contributions of the different periodic orbits out of the spectra and wave functions. The most spectacular manifestation of the periodic orbits is the scarring phenomenon, found in many wave functions of chaotic billiards [13].

References

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