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ANNOUNCEMENT

CAMTP Symposium on Physics

in Honour of Professor Wolfgang Kundt

of University of Bonn, Germany

on occasion of his 80th birthday

Wednesday, 19 October 2011

University of Maribor, Slomškov trg 15, 2000 Maribor

Trstenjakova dvorana

Invited speakers:

Mr. Benjamin Batistić, CAMTP, University of Maribor
Mr. Diego Fregolente Mendes de Oliveira, CAMTP, Univ. of Maribor
Prof. Martin Horvat, University of Ljubljana
Mr. Enej Ilievski, University of Ljubljana
Prof.Dean Korošak, CAMTP and University of Maribor
Prof. Wolfgang Kundt, University of Bonn, Germany
Mr. Georgios Papamikos, CAMTP, University of Maribor
Prof. Tomaž Prosen, University of Ljubljana
Prof. Marko Robnik, CAMTP, University of Maribor
Prof. Valery Romanovski, CAMTP, University of Maribor
Prof. Gregor Veble, Pipistrel, Univ. of Nova Gorica, and CAMTP, UM

PROGRAMME SCHEDULE:

09:45 - 10:00	Opening
10:00 - 11:00	Wolfgang Kundt
11:00 - 11:45	Marko Robnik
11:45 - 12:00	Coffee & Tea
12:00 - 12:45	Tomaz Prosen
12:45 - 13:15	Valery Romanovski
13:15 - 15:00	Lunch break
15:00 - 15:30	Gregor Veble
15:30 - 15:50	Diego Fregolente Mendes de Oliveira
15:50 - 16:10	Georgios Papamikos
16:10 - 16:40	Coffee & Tea
16:40 - 17:10	Martin Horvat
17:10 - 17:30	Benjamin Batistic
17:30 - 17:50	Enej Ilievski
17:50 - 18:20	Dean Korosak
18:30	Dinner

The conference is open for the public free of charge, there is no registration fee, which includes the scientific programme and the services in the coffee breaks.

All interested researchers, also graduate students and undergraduate students, are welcome to attend.

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Below you will find attached the titles and the abstracts of all talks.

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Professor Wolfgang Kundt, University of Bonn, Germany

Professor Wolfgang Kundt was born in Hamburg in 1931. He has received the diploma in theoretical physics at the University of Hamburg in 1956, PhD in 1959 and habilitation in 1965. He was one of the most outstanding students of Professor Pascual Jordan, mainly working with him on theory of General Relativity.

Pascual Jordan (1902-1980) himself is one of the founders and pioneers of quantum mechanics, and together with Max Born and Werner Heisenberg he was co-author of an important series of papers on quantum mechanics. He went on to pioneer early quantum field theory before largely switching his focus to cosmology before World War II. Wolfgang Kundt has also been working on a wide spectrum of problems in theoretical physics, from general relativity, to the the quantum field theory, mathematical physics, cosmology, astrophysics and even biophysics.

Wolfgang Kundt won a professorship in Hamburg in 1971 and since 1977 he was appointed as associate professor at the University of Bonn. He was retired in 1996 and is now professor emeritus at the University of Bonn, still, for his age, extremely active in the research field of his interests. Since 1977 he is mainly working in theoretical astrophysics, but also in biophysics. He is considered to be one of the most original and universal theoretical astrophysisicists, as he has dealt with practically all the most important open problems in theoretical astrophysics, often proposing alternative solutions, not yet accepted or confirmed by the wide scientific community. In this sense he is one of the best theoretical astrophysicists worldwide.

He has spent extended research visits at many places worldwide, such as Syracuse (N.Y., 1959), London (1963), Pittsburgh (Pa, 1966), Edmonton (1971), CERN (1972), Bielefeld (1973), Cambridge (GB, 1977), Kyoto (1978), India (83, 85, 87, 97, 00, 02), Boston (1986), Maribor (since 1991, almost yearly), Linz (1999), Taiwan (2002), Canakkale (2006, 2007), Rio (2008).

He moved into astrophysics via cosmology (since 1972), with interests extending successively into planetary physics, geophysics, and biophysics. He was principal investigator of Helios Experiment II (1969-1979) and director of Erice School on "Neutron Stars, Active Galactic Nuclei, and Jets" (since 1985). He has over 270 publications, among them 6 books. Recently he published a book "Understanding Physics", ed. A.K. Richter (1998), and "Astrophysics, A New Approach", Springer (2005).

His former PhD-students and postdocs are: Hans Juranek, Hajo Leschke, Hans-Jürgen Seifert, Eckhard Krotscheck, Max Camenzind, Marko Robnik, Axel Jessner, Reinhold Schaaf, Hsiang-Kuang Chang, Carsten van de Bruck and Udo Wernick.

Professor Kundt is visiting CAMTP almost every year, each time delivering several seminar lectures on theoretical astrophysics, giving us an opportunity to catch up with the most recent results of observations and theoretical developments from a critical point of view. He also was a visiting professor at the Pedagogical Faculty, Physics Department, of the University of Maribor, in the winter semester 2004/2005, based on a scholarship by the German Government.

I have received my PhD in September 1981 at the University of Bonn under his mentorship. Alltogether I have spent in Bonn more than five years, as PhD student and also as a full time researcher. That period in Bonn was remarkable. I have learned so much from Wolfgang how to do physics efficiently, in many different disciplines, all over theoretical physics and astrophysics. Those extremely productive times in Bonn are unforgetable, especially as the international atmosphere in his research group was extremely pleasant. I wish to thank Wolfgang for all his efforts and support during my scientific Bonn Era.

We are happy to have the opportunity to honour him on occasion of his 80th birthday with this CAMTP Symposium on Physics, and wish him a happy birthday and many happy returns in science and in his private life.

Prof.Dr. Marko Robnik, Director of CAMTP

Fermi acceleration in general time-dependent chaotic billiards

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I will present some new results regarding the properties of the time-dependent chaotic billiards in general. Billiards are simple Hamiltonian systems, defined as a bounded domain in the configuration space, where a point particle is freely moving and collides elastically with the boundary. It is well known that the most of the time-dependent billiards exhibit the interesting phenomenon of an unbounded energy growth, known as the Fermi acceleration [3,4,5,6,7,8]. Fermi [1] and Ulam [2] were the first who considered the time-dependent billiard, as a model, which would explain the origin of large energies of cosmic rays particles by their collisions with the moving interstellar magnetic domains. From the numerical calculations it is well known that the evolution of the particle velocity v follows the power law $v = n^\beta$, with respect to the number of collisions n. The acceleration exponent β, depends strongly on the geometrical properties of the billiard and its time-dependence. It is quite nontrivial to calculate the particular value of β. Recently the value 1/6 for β was calculated for the chaotic conformally breathing billiards [9]. In this talk, the generalization of the ideas in [9], to arbitrary time-dependencies is presented and applied to the chaotic systems, for which the complete set of possible values for the acceleration exponent is derived, $\beta = (0, 1/6, 1/4, 1/2)$. Additionally, I will present how the theory can be used to explain the dissipative time-dependent systems [5,10,11,12]. In particular, I shall explain the numerically discovered scaling relations [5], that connect the saturation velocity, acceleration exponent, and saturation time with the dissipation strength parameter.

References

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Dissipation as a mechanism to suppress Fermi acceleration in time-dependent billiard systems

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We shall present recent results on time-dependent 2D billiard systems. It is well known that the structure of the phase space depends on the individual characteristics of each system. Here, we consider the two types, namely, one integrable (elliptical billiard) [1] and one ergodic (Lorentz gas) [2]. Our main goal is to understand and describe the behaviour of the particle's average velocity (and hence its energy) as a function of the number of collisions considering both, the conservative as well as the dissipative dynamics. For the dissipative case we consider two kinds of dissipation, namely, collisional dissipation and in-flight dissipation. Our results confirm that unlimited energy growth is observed for the non-dissipative case in the two cases. However, when dissipation is introduced via inelastic collisions or in-flight dissipation, the scenario changes and the unlimited energy growth is suppressed, thus leading to a phase transition from unlimited to limited energy growth. The behaviour of the average velocity is described using scaling arguments.

References

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Autonomous relativistic global navigation satellite system

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Global Navigation Satellite Systems (GNSS), such as Galileo and GPS, have to consider distortions caused by the Earth on space and time in its vicinity (space-time curvature) and the effects of relative motions between the spacecrafts and the user (relativistic inertial effects). It uses the Newtonian conception of absolute space and time tied to an average geodetic model and add relativistic corrections depending on the desired accuracy. A new and original approach is to model the navigation system itself directly in the general relativity, where the space and time are not considered as absolute. This approach simplifies the navigation systems and makes it autonomous, which will be in more detail presented in the lecture.

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Nonequilibrium Phase Transition in a Periodically Driven XY Spin Chain

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We present a general formulation of Floquet states of periodically time-dependent open Markovian quasifree fermionic many-body systems in terms of a discrete Lyapunov equation. Illustrating the technique, we analyze periodically kicked XY spin-1/2 chain which is coupled to a pair of Lindblad reservoirs at its ends. A complex phase diagram is reported with reentrant phases of long range and exponentially decaying spin-spin correlations as some of the systems parameters are varied. The structure of phase diagram is reproduced in terms of counting nontrivial stationary points of Floquet quasiparticle dispersion relation.

References

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Spatially embedded complex networks

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Network theory has been successfully used in exploring the structure of many complex systems in the last decade. In the first part of this talk we shall review some general concepts of complex networks with special emphasis on spatial networks.

In the second part of the talk we will, as examples, consider the problem of embedding scale-free networks in fractal space and present the construction of spatial complex networks of pancreatic islet, a compact microorgan in which the release of insulin is under physiological conditions robustly controlled by an efficient cell-to-cell network communication.

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The Interstellar Matter, Cosmic rays, Gamma Ray Bursters, and the Geometry of the Heliosphere

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What exactly are the surroundings of our Solar System? The two Voyager missions have been launched to explore this question, and have crossed the termination shock of the solar wind in Dec.2004, Aug.2007, with several surprises, but new cartoons have not been drawn yet. Apparently, we find ourselves deep inside a quasi-spherical heliosphere, of radius $R \leq 1lyr$, density $n \leq 0.1cm^{-3}$, moving subsonically through a CR-dominated ISM which is dominantly injected by all the Galactic neutron stars.

Time-Dependent Modulations of Linear and Nonlinear Oscillators
Exact and Asymptotic solutions

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I will discuss some topics on time-dependent modulations of the linear and nonlinear oscillators. First, I will briefly treat the general time-dependent quartic (nonlinear) oscillator from the Lie symmetries point of view. Equations that they have at least one Lie point symmetry are obtained and a connection with a reduced first order nonsolvable (in general) equation of the Abel type is established. Next we give some motivation from mathematical physics, namely I make some comments on stationary Schödinger's equation and Einstein's adiabatic invariant of the time-dependent linear oscillator and continue with a description of the classical WKB method for slowly varying (in time) linear oscillators. Some comments and connections with the theory of Lie are given. Finally, I will present a generalisation, which myself and Professor M. Robnik proposed, of the classical WKB method for all time-dependent oscillators with homogeneous power-law potential, i.e.

$$\ddot{q} + a(\epsilon t)q^{2m+1} = 0, \quad 0 < \epsilon \ll 1, \quad m \in \mathbf{N}, \tag{1}$$

where $a(t)$ an arbitrary positive function. The approximation that we obtained turns out to be excellent for suffiecient small $\epsilon \leq 0.5$ but it also captures the behaviour of the real solution for $\epsilon = O(1)$. In the end I shall make a connection of equation (1) with a more general class of nonlinear time-dependent oscillators.

References

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Exact nonequilibrium steady state of a strongly driven open anisotropic Heisenberg chain

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We address the non-equilibrium quantum transport problem in one dimension [1] using the approach of open quantum system and Lindblad equation [2].

In fact, we demonstrate a non-trivial exactly solvable case of the many-body Lindblad equation with strongly correlated bulk Hamiltonian (namely, the anisotropic Heisenberg spin 1/2 chain) and simple dissipation/decoherence (i.e. Lindblad) operators acting on the boundary two spins of the chain only. An exact ladder-tensor-network ansatz is presented [3] for nonequilibrium steady state of the open Heisenberg model in the far from equilibrium regime. We show that the steady-state density operator of a finite system of size n is – apart from a normalization constant – a polynomial of degree $2n-2$ in the coupling constant. Efficient computation of physical observables is facilitated in terms of a transfer-operator reminiscent of a classical Markov process. In the isotropic case we find cosine spin profiles, $1/n^2$ scaling of the spin current, and long-range correlations in the steady state.

Furthermore, the perturbative (weak coupling) version of our ansatz [4] is used to derive a novel pseudo-local conservation law of the anisotropic Heisenberg model, by means of which we rigorously estimate the spin Drude weight (the ballistic transport coefficient) in the easy-plane regime. This closes a long standing question in strongly correlated condensed matter physics.

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Adiabatic invariants and some statistical properties of the time-dependent linear and nonlinear oscillators

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We shall present some recent studies of 1D time dependent linear [1] and some nonlinear oscillators [12]. Our main concern is the accuracy of preservation of the adiabatic invariants and the related question of the statistical properties, such as the time evolution of the energy distribution for the microcanonical ensemble of initial conditions. In case of the linear oscillator this problem can be largely solved rigorously and the solutions can be well approximated using the WKB method, which we develop exactly to all orders [10,1]. It turns out in full generality and rigorously that the adiabatic invariant at the average energy in a linear oscillator never decreases, and remains constant only for ideal adiabatic changes (infinitely slow variation of the parameter of the Hamiltonian). This property is lost in nonlinear oscillators, in general, but reappears for sufficiently fast (nonadiabatic) variation of the Hamiltonian.

In nonlinear oscillators the main problem is that we do not know the phase flow, unlike the linear oscillator case where the flow is linear in phase space. Therefore it is hard to obtain the analytic results, as we must average the relevant quantities (the energy) as a function of final the phase space coordinates with respect to the initial conditions, assuming a microcanonical distribution of the initail conditions. Therefore we use as a model system the time dependent quartic oscillator, for which some analytic results are posssible, whilst many results are obtained by numerical integrations.

While the linear oscillator behaves like explained above, for the quartic oscillator we see that the adiabatic invariant (action) at the average energy can decrease for sufficiently slow (adiabatic) variation of the Hamiltonian, but increases for sufficiently fast changes, in particular in parametric kicks (instantaneous discontinuous jump of the system parameter). The latter result can be rigorously proven, also for other power law potentials, and a universal scaling function for the action is derived. We also study parametrically kicked quartic oscillator, like kick and anti-kick, and periodic kicking. As in this case the Hamiltonian is piecewise constant, we have the rigorous exact analytic phase flow (propagator) in terms of the Jacobi elliptic functions. We study the parametric resonances in the linear and nonlinear oscillator, and their statistical behaviour. In the nonlinear case we find the generic nonlinear picture (regular islands surrounded by chaotic sea), and for the initial conditions in the chaotic sea we observe unbounded energy growth, which is an analogy of Fermi acceleration in 2D time dependent billiard systems.

Our study contributes to the understanding of the statistical behaviour of linear and nonlinear oscillators and of their adiabatic invariants. The adiabatic limit is very important, but also other intermediate cases, and also another extreme, opposite to the adiabatic case, namely the parametric kicking. We predict that in parametric kicks the adiabatic invariant at the average final energy always increases, which is linked to irreversibility in the mean, since the entropy is the logarithm of the adiabatic invariant. We can prove this rigorously for the power law potentials. The understanding of the multiple kicks and their impact on the statistical behaviour of such systems is of great importance in the context of statistical mechanics of low-dimensional Hamiltonian dynamical systems.

References

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Integrability and some bifurcational problems in the theory of ODEs

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Consider a two-dimensional system of ODE's

$$\frac{dx}{dt} = P(x,y), \qquad \frac{dy}{dt} = Q(x,y), \tag{2}$$

and suppose that the coefficients of the polynomials are parameters. In the case when the origin of the system is a non-degenerate center or focus, a limit cycle bifurcates from the origin, when the linearized system changes its stability. This is the well-known Andronov-Hopf bifurcation. The limit cycle bifurcations which depend on nonlinear terms of such system (sometimes such bifurcations are called degenerate Andronov-Hopf bifurcations) are much less investigated, but there is an approach to their study suggested by N.N. Bautin. Bautin also introduced the notion of cyclicity, which nowadays plays an important role in the theory of bifurcations. By the definition, the cyclicity of an elementary focus or center of a polynomial system of ODEs is the maximum number of limit cycles that can be made to bifurcate from the singularity under small perturbation of parameters of the system (the problem of cyclicity of polynomial systems is often called the local 16th Hilbert problem). In the talk we discuss an approach to study the cyclicity problem and the closely related problem of bifurcation of critical periods. We also show their relation to the problem of integrability of plane systems of ODEs and present some methods for finding first integrals of such systems.

References

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Measure stretching exponents and cosmic ray arrival directions

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In a chaotic system, the long term stretching of the local neighbourhood of a trajectory is characterized by the Lyapunov exponent. In an ergodic system, the Lyapunov exponent is independent of the initial point, meaning that long time local stretching rates are equal everywhere. Nevertheless, when observing the stretching of some global object in phase space when evolved under the phase space flow, the long term increase of its measure is not characterized by the Lyapunov exponents but by a set of related stretching exponents that are different and typically larger than the corresponding Lyapunov exponents. I will demonstrate the origin of the discrepancy between the local and global stretching exponents and how it relates to variances in short time stretching rates.

I will then focus on the case of deterministic chaotic random maps. These can be considered as a model for cosmic rays traveling through random magnetic fields. I will demonstrate that the global length stretching exponent is the relevant quantity characterising the number of possible different directions for cosmic rays to reach a point in space (i.e. Earth) when originating from a single source.

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