

---

---

CAMTP

---

---

**”Let’s Face Chaos  
through  
Nonlinear  
Dynamics”**

**7th International**

**Summer School/Conference**

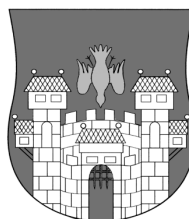


**at the University of Maribor**

**29 June - 13 July 2008**

*Dedicated to the 65th Birthday of Professor Giulio Casati*

**Maribor**



**Slovenia**



---

# Table of Contents

Foreword	2
Programme Schedule	4
Cultural, Social and Touristic Events	6
Organizing Committee	8
Invited Lecturers and Speakers	9
Abstracts of Invited Lectures	20
List of Participants	102
Abstracts of Short Reports	106
Abstracts of Posters	122
Concerts	136
Sponsors	147

# Foreword

The series of by now traditional international Summer Schools/Conferences "Let's Face Chaos through Nonlinear Dynamics" began in the year 1993 on the initiative of a group of undergraduate students of the various faculties at the University of Ljubljana, under the leadership of Mrs. Maja Malus, a student of electrical engineering at the time (now a medical doctor at Harvard), under the scientific guidance of Professors Marko Robnik, Aneta Stefanovska and Igor Grabec. Since 1996 the Schools/Conferences are held exclusively at the University of Maribor, under the organization of CAMTP - Center for Applied Mathematics and Theoretical Physics, every three years. The 7th School/Conference is the largest, according to the number of invited lecturers (49), whilst it is third largest as to the number of other participants (about 38). Also, according to the richness of the scientific and cultural programme, it is probably the best, as you can see in the following pages of this Book of Programme.

The character of our Schools/Conferences is strongly international, we have invited lecturers and participants from all over the world, from almost all continents, and the national component of the Slovenian participants in total (invitees and others) is about 10%. They are strongly interdisciplinary, with the focus on the rich variety of problems in nonlinear science, in mathematics, natural sciences and engineering in the field of chaos, synergetics and theory of complex systems, but physics is by far the most important discipline. As for the scientific level we believe that we are gathering the worldwide leadership and elite, not only among the invited speakers, but also quite pronounced in the other participants, most of them are very talented and productive young scientists from some of the best research groups in the world. So, our gatherings in Maribor have the following dimensions: High level science, internationality, interdisciplinarity, special attention to young students and scientists, promoting them and also helping them financially (especially for those coming from financially weak countries), and finally the cultural dimension which ties together science and life, in a cosmopolitan spirit, without any nationalisms, but rather in mutual respect of all cultures of the world. In fact, it is one of my by now quite popular sayings, that "The Science is the Culture of the World", it is a universal culture indeed, like music.

As the main organizer of the Schools/Conferences, I have made every effort to make your stay in Maribor scientifically as profitable as possible, also to make it culturally as much enjoyable as possible, hoping that you will not only acquire new knowledge, but also successfully present your own research work, and make new scientific collaborative links and creative friendships. This is the most important face of the Maribor gatherings, highly successful and appreciated so far.

I should like also to emphasize the personal component of the 7th School/Conference, namely the fact that it is dedicated to the 65th birthday of Professor Giulio Casati (born on 9 December 1942), from University of Insubria at Como, Italy, (since 2005 also distinguished Professor of the of National University of Singapore), where he is working continuously since the foundation of this university, in which he was involved as the main founder. We feel honoured by his attending of our Schools and Conferences since 1994, by the fact that he accepted our invitation to act as Honorary Director, along with the four other distinguished scientists and colleagues, namely Professors Predrag Cvitanović of Atlanta, Theo Geisel of Göttingen, Siegfried Grossmann of Marburg and Hermann Haken of Stuttgart. Our Schools/Conferences have been dignified by the presence of Professor Giulio Casati, and his lecturing already since the year 1994 (2nd School/Conference). Professor Giulio Casati's contributions to science are immense and brilliant, not only in research, but in the general culture of science. He has organized so many historically important conferences, workshops, schools and other meetings, that it is impossible to list here all of them, and thus he contributed to the evolution of the scientific thought as well as to the evolution of the general social and cultural value of science. In these activities he has always paid a special attention to young people and supported them strongly. He is one of the initiators of the nonlinear science and of the physics of complex systems. His main contributions are in the field of classical and quantum chaos, statistical physics, theory of quantum information and quantum computers, and much more. His invaluable brilliant scientific contributions to this field of theoretical physics are of permanent value and are appreciated by all generations of scientists. We consider it a privilege to celebrate his 65th birthday at our 7th School/Conference, enjoying the opportunity to thank him sincerely for his great work and contributions to science and culture. Science is the universal culture, it is indeed Culture of the World. We look forward to our gathering in Maribor 2008 to cheerfully celebrate Professor Giulio Casati's 65th Birthday and to review his enormous scientific life opus and his current research work. It is on Tuesday 8 July 2008, at 21:00 hours, that we shall start the official celebration of his 65th birthday by a chamber music concert, performed by the young

---

Slovenian cellist Niko Sajko, followed by a celebration and reception in the Kazinska dvorana of the Slovenian National Theatre in Maribor.

But there is yet another personal component. At our gathering we shall remember life and scientific opus of Professor Boris Chirikov of Novosibirsk, Russia, who passed away on 12 February 2008. He was working at the Budker Institute of Nuclear Physics and is the father of classical and quantum chaos in physics. His ideas, results and motivations are brilliant and were very influential worldwide over past four decades, although not yet sufficiently well recognized. He has educated a number of excellent younger scientists who now work at universities worldwide, and was also associated with our schools and conferences, namely he was the main invited lecturer at the 2nd School & Conference in Ljubljana in 1994. Professor Giulio Casati was one of the most important collaborators of Professor Boris Chirikov over the many years. We want to remember Professor Chirikov and pay respect for his great scientific life work.

Last but not least, I should thank all the Members of the international Organizing Committee for their support and help: The Honorary Directors Giulio Casati, Predrag Cvitanović, Theo Geisel, Siegfried Grossmann and Hermann Haken, and the Members: Yoji Aizawa, Tokyo, Tassos Bountis, Patras, Tomaž Prosen, Ljubljana, Valery Romanovski, Maribor, Andreas Ruffing, Munich and Aneta Stefanovska, Lancaster. Among the local organizers my very special thanks go again to coworkers at CAMTP, namely Dr. Janez Kaiser for his help in setting up the home pages, Mr. Benjamin Batistić for lots of technical work in preparing the programme book in which Mr. Gregor Vidmar was also involved.

Our special gratitude must be acknowledged to our respected general sponsors: The Slovenian Research Agency, Ministry of Higher Education, Science and Technology of the Republic of Slovenia, The City of Maribor, represented by the Mayor of the Town, Mr. Franc Kangler, GEN Energija, Nova Kreditna Banka Maribor, TELEKOM Slovenije, Slovenian National Theatre Maribor, VINAG Wine Company, and Festival Lent.

At the very end, thanks go to all participants for coming to Maribor and contributing to a traditionally productive and enjoyable friendly scientific atmosphere. Without the financial and intellectual contribution by the participants, this event would be absolutely impossible.

I wish you all a scientifically successful and culturally pleasant stay in Maribor, and of course, please enjoy the Maribor Festival Lent 2008, the fireworks, and all the cultural programme, the mountains, the excursions and trips, the Slovenian cuisine and wines, the concerts, and the art exhibition, authored by Mrs. Edita Mileta in Kavarna ART of the Hotel PIRAMIDA.

Professor Dr. Marko Robnik  
— Director of **CAMTP** —  
— Director General of the Summer School/Conference —

Maribor, 6 June 2008

1st Week: 30 June - 5 July						
	MONDAY 30 June	TUESDAY 1 July	WEDNESDAY 2 July	THURSDAY 3 July	FRIDAY 4 July	SATURDAY 5 July
Chairman	<i>Robnik</i>	<i>Ott</i>	<i>Hu</i>	<i>Mehlig</i>	<i>Robnik</i>	<i>Chairman: Cvitanović</i>
09:00 - 10:00	<b>Geisel</b>	<b>Takatsuka</b>	<b>Cvitanović</b>	<b>Cvitanović</b>	<b>Takatsuka</b>	<i>Ajisaka</i> 09:00-09:20
10:00 - 11:00	<b>Aizawa</b>	<b>Geisel</b>	<b>Ott</b>	<b>Bäcker</b>	<b>Prosen</b>	<i>Tudorovskiy</i> 09:20-09:40
11:00 - 11:30			- COFFEE & TEA -			<i>Bahraminasab</i> 09:40-10:00
11:30 - 12:30	<b>Dvorak</b>	<b>Tass</b>	<b>Jung</b>	<b>Dvorak</b>	<b>Toda</b>	<i>Bezuglyy</i> 10:00-10:20
12:30 - 13:30	<b>Wilkinson</b>	<b>Harayama</b>	<b>Hasegawa</b>	<b>Wilkinson</b>	<b>Shudo</b>	<i>Hasumi</i> 10:20-10:40
13:30 - 15:15			- LUNCH -			<i>Kenwright</i> 10:40-11:00
Chairman	<i>Aizawa</i>	<i>Haake</i>	<i>Shudo</i>	<i>Romanovski</i>	<i>Takatsuka</i>	- COFFEE & TEA - 11:00-11:30
15:15 - 16:15	<b>Robnik</b>	<b>Tass</b>	<b>Haake</b>	<b>Haake</b>	<b>Robnik</b>	<i>Lippolis</i> 11:30-11:50
16:15 - 16:45			- COFFEE & TEA -			<i>Mendez</i> 11:50-12:10
16:45 - 17:45	<b>Wilkinson</b>	<b>Richter</b>	<b>Haake</b>	<b>Ott</b>	<b>Žnidarič</b>	<i>Mitsui</i> 12:10-12:30
17:45 - 18:45	<b>Mehlig</b>	<b>Repovš</b>	<b>Mehlig</b>	<b>Ott</b>	<b>Shudo</b>	<i>Orihashi</i> 12:30-12:50
	18:45-20:30 - DINNER -	18:45-20:30 - DINNER -	<b>Marhl</b> 18:45-19:15	<i>Mencinger</i> 18:45-19:05	<i>Lhotka</i> 18:45-19:15	<i>Paskauskas</i> 12:50-13:10
	20:30 <b>Welcome, Jazz &amp; Art Exhibition</b>	21:00 <b>Concert</b>	19:15-21:00 - DINNER -	<i>Kutnjak</i> 19:05-19:25	19:15-21:00 - DINNER -	<i>Shinkai</i> 13:10-13:30
		22:00 <b>Reception</b>	19:25-21:00 - DINNER -			- LUNCH - 13:30-15:15
						15:15-16:45 <b>Exc. City of Maribor</b>
						16:45-18:15 <b>Wine tasting</b>
						20:00 <b>Concert &amp; Banquet</b>
						23:45 <b>Fireworks</b>

2nd Week: 7 July - 12 July						
	MONDAY 7 July	TUESDAY 8 July	WEDNESDAY 9 July	THURSDAY 10 July	FRIDAY 11 July	SATURDAY 12 July
Chairman	<i>Wunner</i>	<i>Eckhardt</i>	<i>Prosen</i>	<i>Casati</i>	<i>Flach</i>	<i>Perc</i>
09:00 - 10:00	<b>Cvitanović</b>	<b>Grossmann</b>	<b>Grossmann</b>	<b>Daido</b>	<b>Grossmann</b>	<b>Bountis</b>
10:00 - 11:00	<b>Hu</b>	<b>Casati</b>	<b>Flach</b>	<b>Weidenmüller</b>	<b>Weidenmüller</b>	<b>Prange</b>
11:00 - 11:30	- COFFEE & TEA -					
11:30 - 12:30	<b>Degli Esposti</b>	<b>Stefanovska</b>	<b>Stöckmann</b>	<b>Eckhardt</b>	<b>Eckhardt</b>	<b>Prosen</b>
12:30 - 13:30	<b>Stefanovska</b>	<b>van der Weele</b>	<b>Bountis</b>	<b>Wunner</b>	<b>Bountis</b>	<b>Romanovski</b>
13:30 - 15:15	- LUNCH -					
Chairman	<i>Bountis</i>	<i>Grossmann</i>	<i>Daido</i>	<i>Korošak</i>	<i>Kudo</i>	<i>Grossmann</i>
15:15 - 16:15	<b>van der Weele</b>	<b>Rosenblum</b>	<b>Zakrzewski</b>	<b>Weidenmüller</b>	<b>Prosen</b>	Horvat 15:15-15:45 Andriopoulos 15:45-16:05
16:15 - 16:45	- COFFEE & TEA -					
16:45 - 17:45	<b>Zyczkowski</b>	<b>Stöckmann</b>	<b>Ruffing</b>	<b>Flach</b>	<b>Korošak</b>	Free Time 16:35-20:00
17:45 - 18:45	<b>McClintock</b>	<b>Stöckmann</b>	<b>Schlagheck</b> <b>Pagon</b>	<b>Shakenov</b> <b>Kudo</b>	<b>Véble</b> Luna Acosta	<b>Last Dinner</b> 20:00 <b>Fireworks</b> 23:45
	18:45-20:30 - DINNER -	18:45-20:30 - DINNER -	20:00 Concert and Festive Dinner	Akimoto 18:45-19:05	18:45-20:30 - DINNER -	
	<b>Zavrtanik</b> 21:00-22:00	21:00 Concert		19:05-21:00 - DINNER -		
		22:00 Reception Birthday party Casati				

# Cultural, Social and Touristic Events

## LENT FESTIVAL 2008

During the entire period of our School and Conference there will be the international Festival Lent, offering a very rich variety of performances every evening and every night in the Lent area of Maribor, on the banks by the river Drava (medieval part of the old town). Each participant receives upon his or her arrival a programme brochure and the entrance pass for all performances, free of charge.

## MONDAY 30 JUNE 2008 20:30: WELCOME PARTY, LIFE JAZZ AND OPENING OF THE ART EXHIBITION

On Monday 30 June 2008 at 20:30 (thus after the dinner) we shall gather in the Art Kavarna in the Hotel PIRAMIDA, to get together and to enjoy a glass of fine Slovenian wine and the life jazz music by trio Matjaž Ferk et al. There will be the official opening of an art exhibition with works of a few best Slovenian artists on displa. The exhibition is authored by the galleriest Mrs. Edita Mileta from Maribor. (Most of the works exhibited in the gallery can be actually bought.)

## TUESDAY 1 JULY 2008 21:00: OFFICIAL OPENING OF THE SCHOOL AND CONFERENCE WITH A CONCERT

The official opening of our 7th School and Conference will take place on Tuesday 1 July 2008 at 21:00 in the Kazinska dvorana of the Slovenian National Theatre in Maribor, at Slomškov trg, just next to the Main University Building and the cathedral.

We begin with a chamber music concert (Mrs. Urška Orešič on piano) followed by the addresses of the Director of the Slovenian Research Agency and by a few participants of the School and Conference.

After the speeches there will be a reception with some good Slovenian wines.

## SATURDAY 5 JULY 2008: MARIBOR, WINE TASTING, CONCERT, BANQUET AND FIREWORKS

15:15-16:45 There will be a guided sight seeing tour through the city of Maribor, starting at Grajski trg, just next to Hotel OREL and the City Castle

16:45-18:15 Visit of the (huge) wine cellar of the VINAG Wine Company at Trg svobode 3 (100 meters from Grajski trg) and wine tasting programme

20:00 We gather at the Art Kavarna of the Hotel PIRAMIDA enjoying a glass of champaign

20:00-20:30 Etno jazz life music by a quarttet Matjaž Ferk et al.

20:30-23:45 Banquet

23:45-00:15 Fireworks of the Festival Lent

## SUNDAY 6 JULY 2008: AN EXCURSION THROUGH SLOVENIA

On this day we organize an excursion which starts at 08:30 and we return to the residences in Maribor in the evening.

Lake Bled (in the Alps), Cave of Postojna, old town of Ljubljana, capital of Slovenia. The price is about 100.-EUR per person, which includes everything, also high level lunch and dinner, except for the drinks. This trip is strongly recommended to participants who are visiting Slovenia for the first time. We return to Maribor at about 23:30.

**Please register for the trip and pay in cash to Mr. Benjamin Batistič at latest on Wednesday 2 July.**

## MONDAY 7 JULY 2008 21:00: PUBLIC EVENING LECTURE

At 21:00-22:00 there will be a Public Evening Lecture by **Professor Danilo Zavrtanik** from University of



---

Nova Gorica, Slovenia, entitled **Sources of ultra-high energy cosmic rays**, describing the results and the future plans of **The International Pierre Auger Observatory**.

The lecture will be given in the Lecture Hall of Hotel PIRAMIDA (amphitheatre at the underground level).

**TUESDAY 8 JULY 2008: DEDICATED TO THE 65TH BIRTHDAY OF PROFESSOR GIULIO CASATI, UNIVERSITY OF INSUBRIA, COMO, ITALY**

This day is dedicated to the celebration of the 65th birthday of Professor Giulio Casati.

10:00-11:00 A Festive Public Lecture by Professor Giulio Casati in the amphitheatre of Hotel PIRAMIDA (just the same where we shall have all other lectures).

21:00-22:00 A chamber music concert by cellist Niko Sajko and Piero Malkoč, double bass, about 60 minutes, at Kazinska Dvorana of the Slovenian National Theatre in Maribor, at Slomškov trg, just next to the Main University Building and the cathedral. After the concert there will be addresses and speeches, followed by a reception with good Slovenian wines.

**WEDNESDAY 9 JULY 2008 20:00: CONCERT AND FESTIVE DINNER**

20:00 We gather at the Art Kavarna of the Hotel PIRAMIDA enjoying a glass of champaign

20:00-20:30 Chamber music concert (Barbara Novak, on piano)

20:30-24:00 Festive dinner

**SATURDAY 12 JULY 2008: LAST DINNER AND FIREWORKS**

We shall gather at 20:00 to enjoy the last common informal dinner with some good wines in good atmosphere and shall admire the closing fireworks of the Festival Lent at 23:45-00:15 in the Lent area. Good bye MARIBOR 2008! See you at MARIBOR 2011.

*All events except for the trips on Sunday 6 July are free of charge for all invited lecturers and for other participants of the School and Conference, as they are covered by the conference budget for the local expenses and by the participation fees.*

# Organizing Committee

## Director General and Chairman

**Marko Robnik** (CAMTP, University of Maribor, Slovenia)

## Honorary Directors

**Giulio Casati** (University of Insubria, Italy)

**Predrag Cvitanović** (Georgia Institute of Technology, USA)

**Theo Geisel** (Göttingen, Germany)

**Siegfried Großmann** (Philipps-Universität Marburg, Germany)

**Hermann Haken** (University of Stuttgart, Germany)

## Members

**Yoji Aizawa** (Waseda University, Japan)

**Tassos Bountis** (University of Patras, Greece)

**Tomaž Prosen** (University of Ljubljana, Slovenia)

**Valery Romanovski** (Maribor, Slovenia)

**Andreas Ruffing** (Munich University of Technology, Germany)

**Aneta Stefanovska** (University of Lancaster, England, UK)

---

# Invited Lecturers and Speakers

- **Prof. Dr. Yoji Aizawa**

Department of Applied Physics  
School of Science and Engineering  
Waseda University  
Okubo 3-4-1, Shinjuku-ku  
Tokyo 1690072  
Japan  
<http://www.phys.waseda.ac.jp/aizawa/professor/>  
Email: [aizawa@waseda.jp](mailto:aizawa@waseda.jp)  
Phone: +(81) (3) 3203 2457  
Fax: +(81) (3) 3200 2457

- **Dr. Arnd Bäcker**

Institut für Theoretische Physik  
Technische Universität Dresden  
D-01062 Dresden  
Germany  
<http://www.physik.tu-dresden.de/~baecker>  
Email: [baecker@physik.tu-dresden.de](mailto:baecker@physik.tu-dresden.de)  
Phone: +(49) (0)351 463 32221  
Fax: +(49) (0)351 463 37297

- **Prof. Dr. Tassos Bountis**

Department of Mathematics  
and Center for Research and Applications of Nonlinear Systems  
University of Patras  
Patras 26500  
Greece  
<http://www.math.upatras.gr/~bountis>  
Email: [bountis@math.upatras.gr](mailto:bountis@math.upatras.gr)  
Phone: +(30) (2610) 997381  
Fax: +(30) (2610) 997381

- **Prof. Dr. Giulio Casati**

Center for complex systems  
Physics department  
Insubria University  
Via Valleggio, 11  
22100 Como  
Italy  
<http://scienze-como.uninsubria.it/complexcomo/web-casati.html>  
Email: [giulio.casati@uninsubria.it](mailto:giulio.casati@uninsubria.it)  
Phone: +(39) 031 238 6211  
Fax: +(39) 031 238 6279

- **Prof. Dr. Predrag Cvitanović**  
School of Physics  
Georgia Institute of Technology  
Atlanta, GA 30332-0430  
USA  
<http://www.cns.gatech.edu/~predrag>  
Email: [predrag.cvitanovic@physics.gatech.edu](mailto:predrag.cvitanovic@physics.gatech.edu)  
Phone: +1:404 385-2502  
Fax: +1:404 385-2506
  
- **Prof. Dr. Hiroaki Daido**  
Department of Mathematical Sciences  
Graduate School of Engineering  
Osaka Prefecture University  
Gakuencho 1-1, Naka-ku  
Sakai 599-8531  
Japan  
Email: [daido@ms.osakafu-u.ac.jp](mailto:daido@ms.osakafu-u.ac.jp)  
Phone: +(81) (72) 254 9366  
Fax: +(81) (72) 254 9366
  
- **Prof. Dr. Rudolf Dvorak**  
ADG - AstroDynamicsGroup, Institute for Astronomy  
University of Vienna  
Türkenschanzstrasse 17  
A-1180 Vienna  
Austria  
<http://www.univie.ac.at/adg/dvorak>  
Email: [dvorak@astro.univie.ac.at](mailto:dvorak@astro.univie.ac.at)  
Phone: +(43) (1) 4277 51840  
Fax: +(43) (1) 4277 9518
  
- **Prof. Dr. Bruno Eckhardt**  
Fachbereich Physik  
Philipps-Universität Marburg  
Renthof 6  
35032 Marburg  
Germany  
<http://www.physik.uni-marburg.de/kosy/>  
Email: [bruno.eckhardt@physik.uni-marburg.de](mailto:bruno.eckhardt@physik.uni-marburg.de)  
Phone: +(49)(0) 6421 28 21316  
Fax: +(49)(0) 6421 28 24291
  
- **Prof. Mirko Degli Esposti**  
Dipartimento di Matematica  
Universita' di Bologna  
Piazza di Porta S. Donato 5  
Bologna, 40126  
Italy  
<http://www.dm.unibo.it/~desposti/>  
Email: [desposti@wdm.unibo.it](mailto:desposti@wdm.unibo.it)  
Phone: +(39) (0) 51 2091931  
Fax: +(39) (0) 51 209 4490

- **Dr. Sergej Flach**  
Max-Planck-Institut fuer Physik komplexer Systeme  
Noethnitzer Str. 38  
01187 Dresden  
Germany  
<http://www.pks.mpg.de/~flach>  
Email: [flach@pks.mpg.de](mailto:flach@pks.mpg.de)  
Phone: +49 351 871 2103  
Fax: +49 351 871 2199
  
- **Prof. Dr. Theo Geisel**  
Max-Planck-Institute for Dynamics and Self-Organization  
and Physics Department, University of Göttingen  
Bunsenstrasse 10  
D-37073 Göttingen  
Germany  
<http://www.nld.ds.mpg.de>  
[geisel@nld.ds.mpg.de](mailto:geisel@nld.ds.mpg.de)  
Phone: +(49) (551) 5176 400  
Fax: +(49) (551) 5176 402
  
- **Prof. Dr. Siegfried Grossmann**  
Fachbereich Physik  
Philipps-Universitaet Marburg  
Renthof 6  
D-35032 Marburg  
Germany  
<http://www.physik.uni-marburg.de/>  
Email: [grossmann@physik.uni-marburg.de](mailto:grossmann@physik.uni-marburg.de)  
Phone: +(49) (0) 6421 28 2 2049  
Fax: +(49) (0) 6421 28 2 4110
  
- **Prof. Dr. Fritz Haake**  
Fachbereich Physik  
Universität Duisburg-Essen  
Lotharstr 1  
47048 Duisburg  
Germany  
[http://www.theo-phys.uni-essen.de/tp/ags/haake\\_dir/haake.html](http://www.theo-phys.uni-essen.de/tp/ags/haake_dir/haake.html)  
Email: [fritz.haake@uni-DuE.de](mailto:fritz.haake@uni-DuE.de)  
Phone: +49 203 379 4757  
Fax: +49 203 379 4732
  
- **Dr. Takahisa Harayama**  
Department of Nonlinear Science  
ATR Wave Engineering Laboratories  
2-2-2 Hikaridai  
"Keihanna Science City" Kyoto  
Japan  
<http://www.wel.atr.jp/index-e.html>  
Email: [harayama@atr.jp](mailto:harayama@atr.jp)  
Phone: +(81) (774) 95 1588  
Fax: +(81) (774) 95 1508

- **Prof. Dr. Hiroshi Hasegawa**  
Institute of Quantum Science  
College of Science and Technology  
Nihon University  
Kabda Surugadai 1-8, Chiyoda-ku  
Tokyo 101-8308  
Japan  
[http : // www.phys.cst.nihon-u.ac.jp/ryosikagakuken.html](http://www.phys.cst.nihon-u.ac.jp/ryosikagakuken.html)  
Email: h-hase@mxj.mesh.ne.jp (home)  
Phone: +(81) (3) 3795 9587 (home)  
Fax: +(81) (3) 3795 9587 (home)
  
- **Prof. Bambi Hu**  
Department of Physics  
Hong Kong Baptist University  
Kowloon Tong  
Hong Kong  
Email: bhu@hkbu.edu.hk  
Phone: 852 3411 7029  
Fax: 852 3411 5813
  
- **Prof. Dr. Christof Jung**  
Instituto de Ciencias Fisicas  
Universidad Nacional Autonoma de Mexico  
Av. Universidad 1001  
62251 Cuernavaca  
Mexico  
Email: jung@fis.unam.mx  
Phone: +(52) (777) 3291784  
Fax: +(52) (777) 3291775
  
- **Prof. Dr. Dean Korošak**  
CAMTP - Center for Applied Mathematics and Theoretical Physics  
University of Maribor  
Krekova 2  
SI-2000 Maribor  
Slovenia  
and  
Faculty of Civil Engineering  
University of Maribor  
Smetanova ulica 17  
SI-2000 Maribor  
Slovenia  
<http://www2.arnes.si/~dkoros6/>  
Email: dean.korosak@uni-mb.si  
Phone: +386 2 2294 323  
Fax: +386 2 2524 179
  
- **Dr. Kazue Kudo**  
Ochanomizu University Academic Production  
Ochanomizu University  
Ohtsuka 2-1-1, Bunkyo-ku  
Tokyo 1128610  
Japan  
Email: kudo.kazue@ocha.ac.jp  
Phone: +(81) (3) 5978 5056

- **Prof. Dr. Marko Marhl**  
Department of Physics  
Faculty of Natural Sciences and Mathematics  
University of Maribor  
Koroška cesta 160  
SI-2000 Maribor  
Slovenia  
<http://www.marhl.com>  
Email: [marko.marhl@uni-mb.si](mailto:marko.marhl@uni-mb.si)  
Phone: +(386) (2) 2293 681  
Fax: +(386) (2) 2518 180
  
- **Prof. Dr. Peter V. E. McClintock**  
Department of Physics  
Lancaster University  
Lancaster LA1 4YB  
UK  
<http://www.lancs.ac.uk/depts/physics/staff/pvemc.htm>  
Email: [p.v.e.mcclintock@lancaster.ac.uk](mailto:p.v.e.mcclintock@lancaster.ac.uk)  
Phone: +44-(0)1524-593073  
Fax: +44-(0)1524-844037
  
- **Prof. Dr. Bernhard Mehlig**  
Department of Physics  
Göteborg University  
41296 Göteborg  
Sweden  
<http://www.physics.gu.se>  
Email: [mehlig@fy.chalmers.se](mailto:mehlig@fy.chalmers.se)  
Phone: +(46) (0) 31 772 3452
  
- **Prof. Dr. Edward Ott**  
IREAP, Paint Branch Drive  
University of Maryland  
College Park, MD 20742  
USA  
Email: [edott@umd.edu](mailto:edott@umd.edu)  
Phone: (301)405-5033  
Fax: (301)405-1678
  
- **Prof. Dr. Dušan Pagon**  
Department of Math. & Comp. Sci.  
Faculty of Natural Sci. and Mathematics  
University of Maribor  
Koroška cesta 160  
2000 Maribor  
Slovenia  
[http://www-mat.pfmb.uni-mb.si/clani/dusan\\_pagon.html](http://www-mat.pfmb.uni-mb.si/clani/dusan_pagon.html)  
Email: [dusan.pagon@uni-mb.si](mailto:dusan.pagon@uni-mb.si)  
Phone: +(386) (2) 235 5607  
Fax: +(386) (2) 251 8180

- **Prof. Dr. Matjaž Perc**  
Department of Physics  
Faculty of Natural Sciences and Mathematics  
University of Maribor  
Koroška cesta 160  
SI-2000 Maribor  
Slovenia  
<http://matjazperc.com>  
Email: [matjaz.perc@uni-mb.si](mailto:matjaz.perc@uni-mb.si)  
Phone: +(386) (2) 2293 681  
Fax: +(386) (2) 2518 180
  
- **Prof. Dr. Richard Prange**  
Department of Physics  
University of Maryland  
College Park, MD 20742  
USA  
Email: [prange@umd.edu](mailto:prange@umd.edu)  
Phone: 1-301-405-6154  
Fax: 1-301-314-9465
  
- **Prof. Dr. Tomaž Prosen**  
Department of Physics  
Faculty of Mathematics and Physics  
University of Ljubljana  
Jadranska c. 19  
Ljubljana SI-1000  
Slovenia  
<http://chaos.fiz.uni-lj.si/>  
Email: [tomaz.prosen@fmf.uni-lj.si](mailto:tomaz.prosen@fmf.uni-lj.si)  
Phone: +(386)(1) 4766 578  
Fax: +(386)(1) 2517 281
  
- **Prof. Dr. Dušan Repovš**  
Institute of Mathematics, Physics and Mechanics  
University of Ljubljana  
Jadranska 19  
Ljubljana 1000  
Slovenia  
<http://pef.pef.uni-lj.si/~dusanr/index.htm>  
Email: [dusan.repovs@fmf.uni-lj.si](mailto:dusan.repovs@fmf.uni-lj.si)  
Phone: +(386) (1) 5892 323  
Fax: +(386) (1) 5892 233
  
- **Prof. Dr. Peter H. Richter**  
Institute for Theoretical Physics  
Bremen University  
Otto-Hahn-Allee  
D-28359 Bremen  
Germany  
<http://www-nonlinear.physik.uni-bremen.de/~prichter/>  
Email: [prichter@uni-bremen.de](mailto:prichter@uni-bremen.de)  
Phone: +(49) (421) 218 3680  
Fax: +(49) (421) 218 4869



- **Prof. Dr. Marko Robnik**  
CAMTP - Center for Applied Mathematics and Theoretical Physics  
University of Maribor  
Krekova 2  
SI-2000 Maribor  
Slovenia  
<http://www.camtp.uni-mb.si/>  
Email: [Robnik@uni-mb.si](mailto:Robnik@uni-mb.si)  
Phone: +(386) (2) 2355 350  
Fax: +(386) (2) 2355 360
  
- **Prof. Dr. Valerij Romanovski**  
CAMTP - Center for Applied Mathematics and Theoretical Physics  
University of Maribor  
Krekova 2  
SI-2000 Maribor  
Slovenia  
<http://www.camtp.uni-mb.si/>  
Email: [valery.romanovsky@uni-mb.si](mailto:valery.romanovsky@uni-mb.si)  
Phone: +(386) (2) 2355 361  
Fax: +(386) (2) 2355 360
  
- **PD Dr. Michael Rosenblum**  
Department of Physics  
Potsdam University  
Am Neuen Palais 10  
D-14469 Potsdam  
Germany  
<http://www.agnld.uni-potsdam.de/~mros>  
Email: [MRos@uni-potsdam.de](mailto:MRos@uni-potsdam.de)  
Phone: (+49) (331) 977 1604  
Fax: (+49) (331) 977 1142
  
- **Prof. Dr. Andreas Ruffing**  
Department of Mathematics  
Munich University of Technology  
Boltzmannstrasse 3  
D-85747 Garching  
Germany  
<http://www.-m6.ma.tum.de/~ruffing/>  
Email: [ruffing@ma.tum.de](mailto:ruffing@ma.tum.de)  
Phone: +(49) (89) 289 16826  
Fax: +(49) (89) 289 16837
  
- **PD Dr. Peter Schlagheck**  
Institut für Theoretische Physik  
Universität Regensburg  
93040 Regensburg  
Germany  
<http://www.physik.uni-regensburg.de/forschung/richter/pages/people/PeterSchlagheck.html>  
Email: [peter.schlagheck@physik.uni-regensburg.de](mailto:peter.schlagheck@physik.uni-regensburg.de)  
Phone: +(49) 941 9432033  
Fax: +(49) 943 4382

- **Prof. Dr. Kanat Shakenov**

al-Farabi Kazakh National University  
Faculty of Mechanics and Mathematics  
National Centre of Space Researches and Technologies named after U.M. Sultangazin  
National Space Agency Republic of Kazakhstan  
050054, Almaty, Goethe street 344  
Kazakhstan  
Email: shakenov2000@mail.ru  
Phone: +(7) (727) 2579572  
Fax: +(7) (727) 2579572

- **Prof. Dr. Akira Shudo**

Department of Physics  
Tokyo Metropolitan University  
Minami-Ohsawa, Hachioji-shi  
Tokyo 192-0397  
Japan  
<http://www.sci.metro-u.ac.jp/nonlinear/en/>  
Email: aizawa@waseda.jp  
Phone: +(81) 42-677-2503  
Fax: +(81) 42-677-2483

- **Dr. Aneta Stefanovska**

Department of Physics  
Lancaster University  
Lancaster LA1 4YB, UK  
<http://www.lancs.ac.uk/depts/physics/staff/stefanovska.htm>  
Email: aneta@lancaster.ac.uk  
Phone: +(44) (1524) 592 784  
Fax: +(44) (1624) 844 037

- **Prof. Dr. Hans-Jürgen Stöckmann**

Fachbereich Physik  
der Philipps-Universität Marburg  
Renthof 5  
D-35032 Marburg  
Deutschland  
<http://www.physik.uni-marburg.de/qchaos/>  
Email: stoeckmann@physik.uni-marburg  
Phone: +(49) (6421) 2824137  
Fax: +(49) (6421) 2826535

- **Prof. Dr. Kazuo Takatsuka**

Department of Basic Science  
Graduate School of Arts and Sciences  
University of Tokyo  
Komaba 3-8-1, Meguro-ku  
Tokyo 153-8902  
Japan  
E-Mail kaztak@mns2.c.u-tokyo.ac.jp  
Phone: +81-3-5454-6588  
Fax: +81-3-5454-6588

- **Prof. Dr. Dr. Peter Tass**  
Institute for Neuroscience and Biophysics 3 - Medicine  
Research Center Jülich  
52425 Jülich  
Germany  
<http://www.fz-juelich.de/inb/inb-3/Start/>  
Email: [p.tass@fz-juelich.de](mailto:p.tass@fz-juelich.de)  
Phone: +49 2461/61-2087  
Fax: +49 2461/61-2820
  
- **Prof. Dr. Mikito Toda**  
Department of Physics  
Faculty of Science  
Nara Women's University  
Kita-Uoya-Nishimachi  
Nara 6308506  
Japan  
<http://minnie.disney.phys.nara-wu.ac.jp/~toda/>  
Email: [toda@ki-rin.phys.nara-wu.ac.jp](mailto:toda@ki-rin.phys.nara-wu.ac.jp)  
Phone: +(81) (742) 20 3383  
Fax: +(81) (742) 20 3383
  
- **Prof. Dr. Jacobus P. van der Weele**  
Mathematics Department  
University of Patras  
26500 Patras  
Greece  
<http://www.math.upatras.gr/~weele/>  
Email: [weele@math.upatras.gr](mailto:weele@math.upatras.gr)  
Phone: +(30) (2610) 997457  
Fax: –
  
- **Prof. Dr. Hans Weidenmüller**  
Max-Planck-Institut für Kernphysik  
P.O.Box 103980  
D-69029 Heidelberg  
Germany  
Email: [Hans.Weidenmueller@mpi-hd.mpg.de](mailto:Hans.Weidenmueller@mpi-hd.mpg.de)  
Phone: +(49) (6221) 516291  
Fax: +(49) (6221) 516502
  
- **Prof. Dr. Michael Wilkinson**  
Department of Mathematics and Statistics  
Open University  
Walton Hall  
Milton Keynes  
MK4 1JZ  
England <http://mcs.open.ac.uk/mw987/>  
Email: [m.wilkinson@open.ac.uk](mailto:m.wilkinson@open.ac.uk)  
Phone: +(44) (1908) 659741  
Fax: not used

- **Prof. Dr. Günter Wunner**

Institute of Theoretical Physics 1  
University of Stuttgart  
70550 Stuttgart  
Germany  
<http://www.itp1.uni-stuttgart.de>  
Email: [wunner@itp1.uni-stuttgart.de](mailto:wunner@itp1.uni-stuttgart.de)  
Phone: +(49) (711) 685 64992  
Fax: +(49) (711) 685 54992

- **Professor Jakub Zakrzewski**

Department of Atomic Optics  
Marian Smoluchowski Institute of Physics  
and Mark Kac Complex Systems Research Centre  
Jagiellonian University  
Reymonta 4  
30-059 Krakow  
Poland <http://chaos.if.uj.edu.pl/~kuba> Email: [kuba@if.uj.edu.pl](mailto:kuba@if.uj.edu.pl)  
Phone: +(48) 126635555  
Fax: +(48)126338494

- **Dr. Marko Žnidarič**

Department of Physics  
Faculty of Mathematics and Physics  
University of Ljubljana  
SI-1000 Ljubljana  
Slovenia  
<http://chaos.fiz.uni-lj.si/~znidaricm>  
Email: [marko.znidaric@fmf.uni-lj.si](mailto:marko.znidaric@fmf.uni-lj.si)  
Phone: +(386) (1) 4766 588  
Fax: +(386) (1) 2517 281

- **Prof. Dr. Karol Życzkowski**

Department of Physics  
Jagiellonian University  
ul. Reymonta 4  
30-059 Kraków  
Poland  
<http://chaos.if.uj.edu.pl/~karol/>  
Email: [karol@tatr.if.uj.edu.pl](mailto:karol@tatr.if.uj.edu.pl)  
Phone: +(48) (12) 663 5780  
Fax: +(48) (12) 633 8494

---

# Public Evening Lecture:

## Sources of ultra-high energy cosmic rays

- **Prof. Dr. Danilo Zavrtnik** Laboratory for Astroparticle Physics  
University of Nova Gorica  
Vipavska 13, P.P. 30  
Rožna Dolina  
SI-5000 Nova Gorica  
Slovenia  
<http://www.ung.si/si/o-univerzi/vodstvo/predsednik/>  
Email: [Danilo.Zavrtnik@p-ng.si](mailto:Danilo.Zavrtnik@p-ng.si)  
Phone: +(386) (5) 3315 223  
Fax: +(386) (5) 3315 224

# Abstracts of Invited Lectures

**On the Variety of Recurrence Phenomena in Chaotic Dynamics**  
**-Revisit to Hamiltonian Chaos from Infinite Ergodicity-**

**Yoji Aizawa**

*Department of Applied Physics Advanced School of Science and Engineering Waseda University 3-4-1 Okubo  
Shinjyuku-ku Tokyo, 169-8555, Japan*

Non-stationary recurrence motions are often observed in chaotic systems. Strong intermittency is a typical example which reveals slow dynamics with  $1/f^\nu$  spectra or slow relaxation of correlation functions (Aizawa, 1984, 1989). In non-integrable hamiltonian systems also there appear such long time tails as an universal phenomenon, where it is surmised that sticky regions or stagnant layers surrounding the invariant sets such as KAM tori or Poincaré-Birkhoff tori play an essential role to induce a kind of strong intermittency with non-stationarity of phase-space trajectories (Aizawa *et al.*, 1989). Detailed structures and the recurrent mechanisms in the stagnant layers have not yet been elucidated clearly in generic cases not only of high dimensional systems but also of small degrees systems. In this report, we consider several cases which display essentially different scaling laws in the recurrent time distribution. Especially our main concerns will be paid toward the non-stationary recurrence properties in relation to the infinite measure ergodicity.

In the first part we discuss the phenomenology in high dimensional hamiltonian systems with many degrees of freedom; lattice vibrations and cluster formation, where it is emphasized that the distribution of survival times obeys the Log-Weibull law, and that the universal law is consistent with the Nekhoroshev's estimation of the characteristic time in Arnold diffusion. Another type of recurrent feature with inverse power law is discussed by carrying out with the Mixmaster universe model of Bianchi type IX, where the infinite measure ergodicity is proved in a lower dimensional subdynamics though the Mixmaster model is totally a hamiltonian system with three degrees of freedom (Aizawa, 1997). The inverse power laws in recurrence time distributions are exactly shown in other two dimensional mappings (modified Cat map, multi-baker's map, and Mushroom billiard, etc) (Miyaguchi, 2007).

In the second part of my talk, we discuss the re-injection mechanism in the stagnant layers by use of several one-dimensional maps (modified Bernoulli map, Log-Weibull map, and Ant-lion map (infinite modal singular map)) which reveal non-stationary long time tails and infinite measure ergodicity. Those one-dimensional systems are not exact reduction from the corresponding high dimensional hamiltonian flows, but we expect that the same type of recurrent mechanism might be derived in high dimension cases. We discuss the origin of the universal scaling relations for various classes of recurrent phenomenon. Some theoretical and numerical results obtained from the renewal theories and the large deviation theories are introduced, and the interrelation between the infinite ergodicity and the non-stationary recurrence phenomena is explained. Especially, it is emphasized that the correlation functions and the power spectral density are random variables obeying universal distributions in the non-stationary regime (Akimoto, 2007). From these results quite similar to the hamiltonian chaos, it is conjectured that the infinite measure ergodicity might be hidden behind the multi-ergodic features of the stagnant layers in hamiltonian chaos (Aizawa, 2005).

### References

- Aizawa Y, Murakami C, and Koyama T 1984 *Prog. Theor. Phys. Suppl.* **79** 96  
Aizawa Y 1989 *Prog. Theor. Phys. Suppl.* **99** 149  
Aizawa Y, Kikuchi Y, Harayama T, Yamamoto K, Ota M, and Tanaka K 1989 *Prog. Theor. Phys. Suppl.* **98** 36  
Aizawa Y Koguro N, and Antoniou I *Prog. Theor. Phys.* 1997 **98** 1225  
Aizawa Y 2005 *Advances in Chemical Physics (Ed. S. A. Rice)* **130 Part B** 465  
Miyaguchi T and Aizawa Y 2007 *Phys. Rev. E* **75** 066201  
Miyaguchi T 2007 *Phys. Rev. E* **75** 066215  
Akimoto T and Aizawa Y 2007 *J. Korean Phys. Soc.* **50** 254

# Dynamical tunneling in systems with a mixed phase space

Arnd Bäcker

*Institut für Theoretische Physik, TU Dresden, Germany*

Typical Hamiltonian systems have a mixed phase space consisting of regular islands, which are dynamically separated from the chaotic sea. While classically no transition between those regions is possible, they are quantum mechanically coupled by the process of dynamical tunneling. We derive theoretical predictions for dynamical tunneling rates, which describe the decay of regular states to the chaotic sea. Using an approach based on the introduction of a fictitious integrable system, agreement with numerical data is found for 1D kicked systems [1] and the mushroom billiard [2]. Finally, we will discuss the relevance of a precise knowledge of dynamical tunneling rates for flooding of regular islands [3,4], transport in rough nano-wires [5], and spectral statistics in systems with a mixed phase space.

## References

- [1] A. Bäcker, R. Ketzmerick, S. Löck and L. Schilling:  
*Regular-to-chaotic tunneling rates using a fictitious integrable system*,  
Phys. Rev. Lett. **100** (2008) 104101.
- [2] A. Bäcker, R. Ketzmerick, S. Löck, M. Robnik, R. Höhmann, G. Vidmar, U. Kuhl and H.-J. Stöckmann:  
*Dynamical tunneling in mushroom billiards*,  
Phys. Rev. Lett. **100** (2008) 174103.
- [3] A. Bäcker, R. Ketzmerick and A. Monastra:  
*Flooding of regular islands by chaotic states*,  
Phys. Rev. Lett. **94** (2005) 054102.
- [4] A. Bäcker, R. Ketzmerick and A. Monastra:  
*Universality in the flooding of regular islands by chaotic states*,  
Phys. Rev. E **75** (2007) 066204.
- [5] J. Feist, A. Bäcker, R. Ketzmerick, S. Rotter, B. Huckestein and J. Burgdörfer:  
*Nano-wires with surface disorder: Giant localization lengths and quantum-to-classical crossover*,  
Phys. Rev. Lett. **97** (2006) 116804.



---

# Applications of the GALI Method to Localization in Nonlinear Systems

Tassos Bountis, Thanos Manos and Eleni Christodoulidi

*CRANS - Center for Research and Applications of Nonlinear Systems,  
Department of Mathematics, University of Patras, Patras, Greece  
e-mail address: bountis@math.upatras.gr*

We investigate localization phenomena and stability properties of quasiperiodic oscillations in  $N$  degree of freedom Hamiltonian systems and  $N$  coupled symplectic maps. In particular, we study an example of a parametrically driven Hamiltonian lattice with only quartic coupling terms and a system of  $N$  coupled standard maps. We explore their dynamics using the Generalized Alignment Index (GALI), which constitutes a recently developed numerical method for detecting chaotic orbits in many dimensions, estimating the dimensionality of quasiperiodic tori and predicting slow diffusion in a faster and more reliable way than most other approaches known to date.

## References

- Antonopoulos Ch, Bountis T 2006, *Detecting Order and Chaos by the Linear Dependence Index*, in *ROMAI Journal* **2** (2), 1.  
Christodoulidi H, Bountis T 2006, *Low-Dimensional Quasiperiodic Motion in Hamiltonian Systems*, in *ROMAI Journal* **2**, (2), 37.  
Skokos Ch, Bountis T and Antonopoulos Ch, 2007, *Geometrical Properties of Local Dynamics in Hamiltonian Systems: The Generalized Alignment (GALI) Method*, in *Physica D*, **231**, 30.

# The Stability of Vertical Motion in the N-body Sitnikov Problem

**Tassos Bountis<sup>1</sup> and K. E. Papadakis<sup>2</sup>**

<sup>1</sup>*CRANS - Center for Research and Applications of Nonlinear Systems,  
Department of Mathematics, University of Patras, Patras, Greece  
e-mail address: bountis@math.upatras.gr*

<sup>2</sup>*Department of Engineering Sciences, Division of Applied Mathematics and Mechanics, University of Patras,  
Patras, Greece  
e-mail address: k.papadakis@des.upatras.gr*

We present results about the stability, bifurcations and families of 3D-periodic orbits of the Sitnikov motions in the case of the restricted N-body problem. Here we consider  $\nu=N-1$  equal mass primary bodies which rotate on a circle and the Nth body moves perpendicular to the plane of the primaries. We extend previous work on the four-body problem to the N-body problem, for  $N=5, 9, 15$  and  $25$ . We found that the Sitnikov family, in all cases under consideration, has only one stability interval. For  $N=5, 9, 15, 25$  we have 14, 16, 18, 20 correspondingly, critical Sitnikov periodic orbits from which 3D-families (no longer rectilinear) bifurcate. We have also studied the fascinating question of the extent of bounded dynamics away from the z-axis, taking initial conditions on x,y planes, at constant  $z(0) = z_0$  values, where  $z_0$  lies within the interval of stable rectilinear motions. We also performed a similar study of the dynamics near some members of 3D families of periodic solutions and found, on suitably chosen Poincaré  $(x, \dot{x})$  surfaces, “islands” of ordered motion, while further away from them most orbits become chaotic and eventually escape to infinity.

## References

- Soulis P S, Bountis T and Dvorak R, 2007 *Stability of Motion in the Sitnikov Problem*, in *Cel. Mech. and Dyn. Astron.*, **99**, 129.
- Soulis P S, Papadakis K E and Bountis T, 2008 *Periodic Orbits and Bifurcations in the Sitnikov Four-body Problem*, in *Cel. Mech. and Dyn. Astron.*, 2008, to appear.
- Skokos Ch, 2001: *Alignment Indices: a New, Simple Method for Determining the Ordered or Chaotic Nature of Orbits*, in *J. Phys. A: Math. Gen.*, **34**, 10029.

# Dynamical Properties and Synchronization of Complex Nonlinear Equations for Detuned Lasers

Tassos Bountis<sup>1</sup> and G. M. Mahmoud<sup>2</sup>

<sup>1</sup>*CRANS - Center for Research and Applications of Nonlinear Systems,  
Department of Mathematics, University of Patras, Patras, Greece  
e-mail address: bountis@math.upatras.gr*

<sup>2</sup>*Department of Mathematics, Faculty of Science, Assiut University, Assiut 71516, Egypt, e-mail  
address: gmahmoud@aun.edu.eg*

We study the dynamics and synchronization properties of a system of complex nonlinear equations describing detuned lasers. These equations possess a whole circle of fixed points, while the corresponding real variable equations have only isolated fixed points. Studying the stability of their equilibrium points, we determine conditions under which the complex equations have positive, negative or zero Lyapunov exponents and chaotic, quasiperiodic or periodic attractors for a wide range of parameter values. We investigate the synchronization of chaotic solutions of our detuned laser system, using as a drive a similar set of equations and applying the method of global synchronization. We find attractors whose 3-dimensional projection is not at all similar to the well-known shape of the (real) Lorenz attractor. Finally, we apply complex periodic driving to the electric field equation and show that the model can exhibit a transition from chaotic to quasiperiodic oscillations. This leads us to the discovery of an exact periodic solution, whose amplitude and frequency depend on the parameters of the system. Since this solution is stable for a wide range of parameter values, it may be used to control the system by entraining it with the applied periodic forcing.

## References

- Mahmoud G M and Bountis T 2004, *The Dynamics of Systems of Complex Nonlinear Oscillators: A Review*, in *Int. J. of Bifurcation and Chaos*, Vol.14(11), 3821-3846.
- Mahmoud G M, Bountis T and Mahmoud E E 2007, *Active Control and Global Synchronization of Complex Chen and Lü Systems*, in *Int. J. of Bifurcation and Chaos*, Vol. 17 (12),4295-4308.
- Mahmoud G M, Aly S A and Al-Kashif M A 2008, *Dynamical Properties and Chaos Synchronization of a New Chaotic Complex Nonlinear System*, in *Nonlinear Dynamics*, to appear.

# Classical and quantum transport: from Fourier law to thermoelectric efficiency

Giulio Casati

*Center for nonlinear and complex systems,  
Universita' della Insubria, Como, Italy*

The understanding of the underlying dynamical mechanisms which determines the macroscopic laws of heat conduction is a long standing task of non-equilibrium statistical mechanics. A better understanding of such mechanism may also lead to potentially interesting applications based on the possibility to control the heat flow. Of particular interest is the problem, almost completely unexplored, of the derivation of Fourier law from quantum dynamics. To this end we discuss heat transport in a model of a quantum interacting spin chain and we provide clear numerical evidence that Fourier law sets in above the transition to quantum chaos. Finally we consider the transport of particles and heat in open classical ergodic billiards. We show that thermoelectric efficiency can approach the Carnot limit for sufficiently complex charge carrier molecules.

## References

- M. Terraneo, M. Peyrard and G. Casati,: " Controlling the energy flow in nonlinear lattices: a model for a thermal rectifier" *Phys. Rev. Lett.* **88**, 094302 (2002)
- G. Casati, and T. Prosen: " Anomalous Heat Conduction in a Di-atomic One-Dimensional Ideal Gas " *Phys Rev E.* **67**, 015203 (2003)
- Baowen Li, G. Casati, and Jiao Wang: "Heat conductivity in linear mixing systems " *Phys Rev E.* **67**, 021204 (2003).
- Baowen Li, Giulio Casati, Jiao Wang, and Tomaz Prosen "Fourier law in the alternate mass hard-core potential chain". *Phys. Rev. Lett.* **92**, 254301 (2004)
- Baowen Li, Lei Wang, and Giulio Casati: "Thermal Diode: Rectification of heat flux". *Phys. Rev. Lett.* **93**, 184301 (2004)
- G. Casati,"Controlling the heat flow: now it is possible" *Chaos* **15**, 015120 (2005)
- C. Mejia-Monasterio, T. Prosen and G. Casati, "Fourier's Law in a Quantum Spin Chain and the Onset of Quantum Chaos" *Europhysics letters* **72**, 520 (2005)
- Baowen Li, Lei Wang, and Giulio Casati: "Negative differential thermal resistance and thermal transistor". *Appl. Phys. Lett.* **88**, 143501 (2006)
- Giulio Casati, C. Mejia-Monasterio and T. Prosen, " Magnetically Induced Thermal Rectification " *Phys. Rev. Lett.* **98**, 104302 (2007)
- G. Casati,C. Mejia-Monasterio, and T. Prosen "Increasing thermoelectric efficiency: a dynamical systems approach" *Phys. Rev. Lett* (in press).

---

# Lecture 1: Hopf's dynamical vision of turbulence

Predrag Cvitanović

*School of Physics, Georgia Tech  
Atlanta, GA 30332-0430, USA*

As a turbulent flow evolves, every so often we catch a glimpse of a familiar pattern. For any finite spatial resolution, the system follows approximately for a finite time a pattern belonging to a finite alphabet of admissible patterns. In “Hopf’s vision of turbulence,” the long term turbulent dynamics is a walk through the space of such unstable patterns.

In this 3-lecture mini-course we start with a recapitulation of basic notions of dynamics; flows, maps, local linear stability, heteroclinic connections, qualitative dynamics of stretching and mixing and symbolic dynamics.

In lecture 2 we discuss the discrete and continuous symmetries of 1- and 3-dimensional fluid flows, and in lecture 3 we illustrate the dynamical approach by moderate  $Re$  boundary sheer turbulence in the plane Couette flow.

## References

J.F. Gibson *et al.*, “Movies of plane Couette,” [ChaosBook.org/tutorials](http://ChaosBook.org/tutorials)

E. Hopf, “A mathematical example displaying features of turbulence,” *Commun. Appl. Math.* **1**, 303 (1948)

B. Hof, C.W.H. van Doorne, J. Westerveel, F.T.M. Nieuwstad, H. Faisst, B. Eckhardt, H. Wedin, R.R. Kerswell and F. Waleffe, “Experimental demonstration of travelling waves in pipe flow,” *Science* **305**, 1594 (2004)

P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay *et al.*,  
*Chaos: Classical and Quantum*, [ChaosBook.org](http://ChaosBook.org)

## Lecture 2: Symmetries and dynamics

Predrag Cvitanović

*School of Physics, Georgia Tech  
Atlanta, GA 30332-0430, USA*

Dynamical systems often come equipped with discrete symmetries, such as the reflection symmetries of various potentials. Symmetries simplify the dynamics in a rather beautiful way: If dynamics is invariant under a set of discrete symmetries  $G$ , the state space  $\mathcal{M}$  is *tiled* by a set of symmetry-related tiles, and the dynamics can be reduced to dynamics within one such tile, the *fundamental domain*  $\mathcal{M}/G$ . If the symmetry is continuous the dynamics is reduced to a lower-dimensional desymmetrized system  $\mathcal{M}/G$ , with “ignorable” coordinates eliminated (but not forgotten). In either case families of symmetry-related full state space cycles are replaced by fewer and often much shorter “relative” cycles. In presence of a symmetry the notion of a prime periodic orbit has to be reexamined: it is replaced by the notion of a relative periodic orbit, the shortest segment of the full state space cycle which tiles the cycle under the action of the group. Furthermore, the group operations that relate distinct tiles do double duty as letters of an alphabet which assigns symbolic itineraries to trajectories.

### Reference

Read chapter “World in a mirror,” of P. Cvitanović, R. Artuso, R. Mainieri, G. Vattay et al., *Classical and Quantum Chaos*, [ChaosBook.org](http://ChaosBook.org)

R. Gilmore and C. Letellier, *The Symmetry of Chaos* (Oxford U. Press, Oxford 2007)

---

## Lecture 3: Geometry of boundary shear turbulence - a stroll through 61,506 dimensions

J. F. Gibson, P. Cvitanović and J. Halcrow

*School of Physics, Georgia Tech  
Atlanta, GA 30332-0430, USA*

In the world of moderate Reynolds number, everyday turbulence of fluids flowing across planes and down pipes a velvet revolution is taking place. Experiments are almost as detailed as the numerical simulations, DNS is yielding exact numerical solutions that one dared not dream about a decade ago, and dynamical systems visualization of turbulent fluid's state space geometry is unexpectedly elegant.

We shall take you on a tour of this newly breached, hitherto inaccessible territory. Mastery of fluid mechanics is no prerequisite, and perhaps a hindrance: the lecture is aimed at anyone who had ever wondered why - if no cloud is ever seen twice - we know a cloud when we see one? And how do we turn that into mathematics?

### References

J.F. Gibson *et al.*, "Movies of plane Couette," [ChaosBook.org/tutorials](http://ChaosBook.org/tutorials)

J.F. Gibson, J. Halcrow and P. Cvitanović, "Visualizing the geometry of state space in plane Couette flow," *J. Fluid Mech.* (2008), to appear; [arXiv:0705.3957](https://arxiv.org/abs/0705.3957)

A. Schmiegel, *Transition to turbulence in linearly stable shear flows*, PhD thesis, Philipps-Universität Marburg (1999), available on [archiv.ub.uni-marburg.de/diss/z2000/0062/](http://archiv.ub.uni-marburg.de/diss/z2000/0062/)

B. Eckhardt, A. Schmiegel, H. Faisst and T. Schneider "Dynamical systems and the transition to turbulence in shear flows," *Phil. Trans. R. Soc (London)* (2005) P. Cvitanović, E. Siminos and R. L. Davidchack, "State space geometry of a spatio-temporally chaotic Kuramoto-Sivashinsky flow," *SIAM J. Applied Dynam. Systems*, submitted; [arXiv:0709.2944](https://arxiv.org/abs/0709.2944)

# Dynamics of a large population of coupled active and inactive oscillators

Hiroaki Daido

*Department of Mathematical Sciences, Graduate School of Engineering,  
Osaka Prefecture University, Sakai 599-8531, Japan*

Coupled nonlinear oscillators governed by dissipative dynamics have been studied extensively in the past decades. Such a system is composed of either limit-cycle oscillators or chaotic oscillators. In particular, much attention has been paid to a variety of interesting phenomena observed in a large ensemble of such oscillators, e.g. synchronization transition and clustering. Results of these studies are of significance in diverse areas of science and technology.

However, there is one point overlooked in such studies, which is the fact that real coupled oscillators, like any other systems, suffer from some kind of deterioration from the beginning or as time passes. Motivated by this, we here consider the effect of "bad components" on the behavior of a population of coupled oscillators, where a "bad component" means an oscillator which has lost the ability of performing self-sustained oscillation. Such a component of the population will be called an "inactive" oscillator, while a component keeping that ability an "active" one. More specifically, we examine what happens in such a system as the ratio of inactive elements  $p$  as well as the coupling strength  $K$  are varied. This problem is important, for example, in understanding the robustness of diverse biological rhythms and in technological contexts, where no system is allowed to be fragile to defects.

After an introduction, we start from some mathematical models of globally and diffusively coupled oscillators which are either periodic or chaotic. Here, we encounter such phenomena as aging transition and desynchronization or clustering. We then proceed to the case of locally coupled oscillators to examine what new features the locality of coupling brings to the system.

## References

- Daido H and Nakanishi K 2004 *Phys. Rev. Lett.* **93** art. no. 104101  
Nakanishi K and Daido H 2006 *Prog. Theor. Phys. Suppl.* **161** pp173-176  
Daido H and Nakanishi K 2006 *Phys. Rev. Lett.* **96** art. no. 054101  
Pazó D and Montbrió E 2006 *Phys. Rev. E* **73** art. no. 055202  
Daido H 2007 *Nonlinear Phenomena in Complex Systems* **10** pp72-78  
Daido H and Nakanishi K 2007 *Phys. Rev. E* **75** art. no. 0562067; **76**, art. no. 049901(E)  
Daido H to be published



# Entropy, Information and Dynamical Systems: mathematical results and applications

Mirko Degli Esposti

*Department of Mathematics  
University of Bologna, Bologna, Italy*

*The discovery that the amount of information in a message (or in any other structure) can be objectively measured was certainly one of the major scientific achievements of the 20th century* (Peter Grassberger)

We shall first review the very basic definition of the *information theoretic entropy* for symbolic sequences, discussing its main mathematical properties and its close relationship to thermodynamic entropy. Then we like to discuss the importance of information entropy for chaotic systems and to mention the role that entropy plays in Bayesian inference (*maximum entropy principle*).

As we will see, mostly through simple examples, estimating the entropy of a message (text document, picture, piece of music or biological signal) is quite important because it gives a measure of its *compressibility*, i.e. the optimal achievement for any possible compression algorithm. The very basic definitions and classical results of coding theory will give us the opportunity to understand in simple set up the exact relationship between entropy and compressibility (*Shannon's (noiseless) coding theorem*).

We then turn to *relative entropy* (also called *Kullback-Leibler divergence*) that allow us to measure statistical differences between two distinct sources of information. Our main objective is to discuss its main mathematical properties and how this quantity can be approximated in practice through a suitable use of everyday (e.g. *winzip* or *gzip*) compression algorithm (*Merhav-Ziv's Theorem*).

After a brief presentation of other recent entropy indicators, together with their applications to sequences generated by chaotic dynamical systems, we conclude discussing how these ideas can be applied to more concrete situations, such as the problem of *authorship attribution* for literary text and automatic classification of cardiac signals.

## References

- Benedetto D, Caglioti E, Loreto V 2002 *Phys. Rev. Lett.* **88**, 048702 (2002)  
 Benedetto D, Caglioti E Gabrielli D, 2006 *J. Stat. Mech.* P09011  
 T. Cover and J. Thomas, *Elements of information theory*, Wiley, New York, 1991.  
 Degli Esposti M, Farinelli C, Menconi G 2007 *Chaos and Fractal*  
 Degli Esposti M, Farinelli C , Manca M , Tolomelli A 2007 *Journal of Biostatistics* **1**, 53-78  
 Grassberger P 2002 arXiv:physics/0207023v1  
 Haixiao Cai, Kulkarni S.R., Verdu, S. 2004 *IEEE Transactions on Information Theory*, **50**, No. 7, pp. 1551-1561  
 Lempel A and Ziv J, 1977 *IEEE Trans. Inf. Th.* 337-343.  
 Merhav N, Ziv J, 1993 *IEEE Trans. Inf. Th.* **39** 1280  
 Puglisi A, Benedetto D, Caglioti E , Loreto V, Vulpiani A 2003 *Physica. D* **180**, no1-2, pp. 92-107  
 C.E. Shannon, 1948 *The Bell System Technical J.* **27** 623  
 Shields P 1988 *IEEE Transactions on Information Theory*, **44**, 2079-2073  
 Shields P 1996 *The ergodic theory of discrete sample paths*, AMS Graduate Studies in Mathematics, American Mathematics Society

# Multiplanetary Solar Systems – A Challenge for Astronomy

Rudolf Dvorak

*University of Vienna, Inst. for Astronomy, AstroDynamicsGroup*

We shall review the structure of the known solar systems with more than one planet. To be able to compare our own system with the known more than 200 extrasolar planetary system (EPS) we discuss the dynamical structure of our planets with this interesting separation in an inner system with the four terrestrial planets and the outer one with the four gasgiants. We also show the new results concerning the longterm stability (unstability?) over billions of years.

We then discuss the most interesting EPS which are stable although some of them they suffer from quite large eccentric orbits. In addition we show results of the possible additional – namely terrestrial – planets which could be interesting from the point of view of searching for other earthlike planets.

## References

Dvorak R, Freistetter F and Kurths J (eds.) 2005 *Chaos and Stability in Planetary Systems*, Lecture Notes in Physics, vol. 683, (Springer) 279 pages

Dvorak R (ed.) 2007 *Extrasolar Planets. Formation, Detection and Dynamics*, (Wiley-VCH) 287 pages

---

# Multiple ionization in strong laser fields

Bruno Eckhardt

*Fachbereich Physik, Philipps-Universität Marburg, 35032 Marburg, Germany*

Strong field multiple ionization has puzzled scientists, among other things, because of the strong correlations among outgoing electrons: their momenta parallel to the polarization axis of the field are very similar. In an effort to explain this process, we have studied the classical dynamics of two electrons escaping from an attracting nucleus. This helps to understand the origin of the correlations and suggests a simplified model that captures the essential process and can be simulated in full quantum calculations.

We find that the repulsion between electrons amplifies deviations from a symmetric escape and thereby enforces the correlates escape between the electrons. This observation can be exploited to describe many of the observed properties, to derive a nonlinear threshold law near the onset of double ionization, and to develop a simple 1+1-dimensional model for effective classical and quantum mechanical simulations. Quantum simulations of this model show good agreement with the classical model, but also reveal fluctuations, which semiclassically can be understood as interferences from multiple paths leading to double ionization.

The dynamics of the correlations can easily be extended to simultaneous escape of more than two electrons, and to multiple ionization in molecules.

## References

- Phys. Rev. A **63**, 043414 (2001)
- Phys. Rev. A **64**, 053401 (2001)
- Europhys. Lett. **56**, 651–657 (2001)
- J. Phys. B: At. Mol. **36**, 3923–3935 (2003)
- Phys. Rev. A **71**, 033407 (2005)
- J. Phys. B **39**, 3865 (2006)
- Phys. Rev. Lett. **98**, 203002 (2007)
- Phys. Rev. A **77**, 015402 (2008)

# Reversibility in many body physics

Bruno Eckhardt

*Fachbereich Physik, Philipps-Universität Marburg, 35032 Marburg, Germany*

The classical equations of motion for particles are reversible in that the trajectories are retraced if the velocities and the direction of time are reversed. This holds true independent of whether the motion is integrable or chaotic. The difference is noticed, however, in the presence of noise or stochastic forces: then the variances increase algebraically in the case of an integrable motion and exponentially in the case of a chaotic motion.

GI Taylor has demonstrated the reversibility in the integrable case for a viscous fluid between rotating concentric cylinders, Chaiken et al has demonstrated the increased stretching in the case of chaotic motions. A classical analysis is given in Eckhardt (2003).

In a twist to these experiments, Pine et al have recently replaced the colored fluid in these experiments by micron-sized particles. Because of the small Reynolds numbers involved the dynamics is overdamped and the dynamics very much dissipative. They noticed that after several oscillations the system self-organizes into one of two kinds of states: (i) for small amplitudes the particles arrange themselves such that they avoid interactions. In this state the dynamics is reversible. (ii) for larger amplitudes the particles cannot avoid interactions and the dynamics is irreversible.

We will discuss the equations of motion for a classical particle in a liquid, present simulations that show the kinds of transitions between the states and discuss evidence for the phase transitions and its critical exponents.

## References

- L. Corte, PM Chaikin, JP Gollub and DJ Pine, *Nature Physics* **4**, 420 - 424 (2008).
- B. Eckhardt, *Journal of Physics A: Mathematical and General* **36**, 371 (2003).
- H. Hinrichsen, *Adv. Phys.* **7**, 815 (2000).
- G.M. Homsy et al, *Multi-media Fluid Mechanics CD*, Cambridge University Press 2004
- D.J. Pine, J.P. Gollub, J.F. Brady, and A.M. Leshansky, *Nature* **438**, 997 (2005).

for movies, see <http://www.physics.nyu.edu/pine/research/hydroreverse.html>

---

# Periodic orbits, localization in normal mode space, and the Fermi-Pasta-Ulam problem

Sergej Flach

*Max Planck Institute for the Physics of Complex Systems  
Nöthnitzer Str. 38, 01187 Dresden, Germany*

In 1955 Fermi, Pasta and Ulam (FPU) reported on the nonequipartition of a nonlinear atomic chain, with initially one normal mode excited. Modern computational studies show, that on a first, relatively short, time scale the energy is distributed among a few neighbouring modes in modal space, with more distant modes being exponentially weakly excited - i.e., one observes localization in normal mode space. On a much larger second time scale (which was not reachable with the MANIAC I), the tail modes are slowly growing, and finally the system does equilibrate. Despite its strong impact on nonlinear dynamics and statistical physics, the paradox remained essentially unexplained for decades.

Recent studies show that the model allows for exact time-periodic solutions (q-breathers), which are exponentially localized in the space of normal modes. The trajectory initially computed by FPU is a slight perturbation away from an exact q-breather orbit. Consequently most of the key observations related to the FPU problem (localization of energy in normal mode space for long times, recurrence on relatively short times, system size and energy thresholds) are captured by the properties of q-breathers and the phase space flow nearby. The underlying concept is much more general, and can be easily extended to two- and three-dimensional finite lattices. In particular, localization properties of q-breathers are shown to depend on intensive control parameters (energy density, wave vectors) ONLY.

## References

- S. Flach et al, *Am. J. Phys.* **76**, 453 (2008).
- K.G. Mishagin et al, arXiv:0801.1055v1 [nlin.PS].
- S. Flach and A. Ponno, *Physica D* **237**, 908 (2008).
- T. Penati and S. Flach, *Chaos* **17**, 023102 (2007).
- S. Flach et al, *Int. J. Mod. Phys. B* **21**, 3925 (2007).
- O. I. Kanakov et al, *Phys. Lett. A* **365**, 416 (2007).
- M.V.Ivanchenko et al, *Phys. Rev. Lett.* **97**, 025505 (2006).
- S. Flach, M.V.Ivanchenko and O.I.Kanakov, *Phys. Rev. E* **73**, 036618 (2006).
- S. Flach, M. V. Ivanchenko and O. I. Kanakov, *Phys. Rev. Lett.* **95**, 064102 (2005).

# Localization Versus Delocalization in Nonlinear Disordered Systems

Sergej Flach

*Max Planck Institute for the Physics of Complex Systems  
Nöthnitzer Str. 38, 01187 Dresden, Germany*

Linear disordered systems allow for Anderson localization. Then all eigenstates are localized, and an initially localized wave packet will not spread beyond the localization volume of the linear system. Adding nonlinearities leads to an interaction of eigenstates. The question is therefore, whether the wave packet will spread or stay localized. I will show that the wave packet can spread in three different regimes. All of them show up with subdiffusion, while some allow also for partial localization due to selftrapping. The subdiffusive spreading is universal and characterized by the second moment growing algebraically in time, with exponent  $1/3$  for one-dimensional systems with cubic nonlinearity. It is due to a finite number of modes which stay resonant and are responsible for weak chaos inside the packet.

I will generalize to other types of nonlinearity, higher spatial dimensions, address the spreading in the presence of a nonzero thermal background, and the corresponding quantum case.

## References

- S. Komineas, G. Kopidakis, S. Flach and S. Aubry, *Phys. Rev. Lett.* **100**, 084103 (2008).  
S. Flach, S. Komineas, D. Krimer and Ch. Skokos, in preparation.

---

# Self-Organized Criticality in Neuronal Systems\*

Theo Geisel

*Max Planck Institute for Dynamics and Self-Organization  
& Physics Department, University of Göttingen  
& Bernstein Center for Computational Neuroscience  
D - 37073 Göttingen, Germany*

Self-organized criticality is one of the key concepts to describe the emergence of complexity in natural systems. The concept asserts that a system self-organizes into a critical state where system observables are distributed according to a power law. It has long been speculated that this phenomenon might also show up in neuronal networks, but so far no genuinely neuronal model has been shown to exhibit full self-organized criticality.

Here we consider a network of integrate-and-fire neurons with depressive dynamical synapses, i.e. where the synaptic coupling exhibits fatigue under repeated presynaptic firing [1]. We find self-organized critical avalanches and show that in a range of interaction parameters this adaptation mechanism drives the network into a self-organized critical regime by adjusting the average coupling strengths to a critical value. We derive an analytic expression for the mean synaptic strengths and the average inter-spike intervals in a mean-field approach. These mean values obey a self-consistency equation which allows us to characterize the self-organization mechanism. Our theory explains recent experimental results, where neuronal avalanches were observed in multi-electrode recordings of cortical slice cultures [2].

## References

- [1] A. Levina, J. M. Herrmann, and T. Geisel, Dynamical synapses causing self-organized criticality in neural networks, *Nature Physics*, 3, 857 (2007).
- [2] J. Beggs, and D. Plenz, Neuronal avalanches are diverse and precise activity patterns that are stable for many hours in cortical slice cultures, *J. Neurosci.* 24, 5216-5220 (2004)

\* work in collaboration with A. Levina and M. Herrmann

# Levydemics\*

Theo Geisel

*Max Planck Institute for Dynamics and Self-Organization & Physics Department, University of Göttingen  
& Bernstein Center for Computational Neuroscience  
D - 37073 Göttingen, Germany*

The efficiency of epidemic modelling and forecasts has suffered in the past from a poor description of the spatial dynamics. Accurate models are needed e.g. to test potential strategies to control the spread of an epidemic. While the local infection dynamics is well understood for many diseases, very little was known about the statistical laws by which humans and their germs disperse. We have tried to improve these models by introducing Lévy processes, which allow for superdiffusive spatial dynamics and have carried out an experiment to verify their applicability.

How can we obtain reliable information on travelling statistics, if people can travel using very different means of transportation from bikes to planes? We have studied this problem empirically using the dispersal of dollar bills as a proxy. The time dependent probability density obtained in this way exhibits pronounced spatiotemporal scaling and anomalous diffusion, which mathematically are described very accurately by our model in terms of a bifractional diffusion equation with few parameters.

## References

- D. Brockmann, L. Hufnagel, and T. Geisel, "The scaling laws of human travel", *Nature* 439, 462 (2006).  
L. Hufnagel, D. Brockmann, and T. Geisel, "Forecast and control of epidemics in a globalized world", *PNAS*, 101, 15124 (2004).

\* work in collaboration with D. Brockmann and L. Hufnagel



---

# Thermal convection beyond the Oberbeck-Boussinesq simplification

## I

### The phenomenon

Siegfried Grossmann

*Fachbereich Physik, Philipps-Universität Marburg, Renthof 6, D-35032 Marburg, Germany*

These three lectures are based on joint work with the following co-authors:

**Detlef Lohse, Francisco Fontenele Araujo, Guenter Ahlers, Eric Brown, Denis Funfschilling, Kazuyasu Sugiyama, Enrico Calzavarini, and Alexander Esser**

An introductory presentation of the basic physics of thermal convection in Rayleigh-Bénard geometry between a warmer bottom plate and a colder top plate is given. The heat current  $Q$  or Nusselt number  $Nu = Q/(\kappa\Delta L^{-1})$  and the amplitude  $U$  or Reynolds number  $UL/\nu$  of the convective flow are considered as functions of the external control parameters Rayleigh number  $Ra = \beta g L^3 \Delta / (\nu \kappa)$  and Prandtl number  $Pr = \nu / \kappa$ . Recent experimental progress is discussed in terms of a unified theory (GL 2000, and subsequent papers), based on the exact equations of motion.

Particular emphasis is put on a currently explored phase space range of very small  $Pr$  numbers (cf. Grossmann & Lohse 2008). It turns out to be strongly turbulent but rather inefficient in heat transport: though  $Re$  increases considerably, the Nusselt number decreases,  $Nu \rightarrow 1$ , i.e., the heat transport becomes mainly conductive.

#### References

- Grossmann S and Lohse D 2000 *J Fluid Mech* **407** 27
- Grossmann S and Lohse D 2001 *Phys Rev Lett* **86** 3316
- Grossmann S and Lohse D 2002 *Phys Rev E* **66** 016305
- Grossmann S and Lohse D 2003 *J Fluid Mech* **486** 105
- Grossmann S and Lohse D 2004 *Physics of Fluids* **16** 4462
- Ahlers G, Grossmann S and Lohse D 2009 *Rev Mod Phys* to appear
- Grossmann S, Lohse D 2008 to be published

# Thermal convection beyond the Oberbeck-Boussinesq simplification

## II

### Physical signatures of non-OB

Siegfried Grossmann et al.

*Fachbereich Physik, Philipps-Universität Marburg, Renthof 6, D-35032 Marburg, Germany*

An experimental puzzle, measuring different  $Nu$  versus  $Ra$  for seemingly the same systems prompted the discussion how to eliminate systematic errors: side wall leakage, plate resistance corrections, or deviations from the Oberbeck-Boussinesq approximation. Significant recent progress has in particular been obtained in measuring and interpreting the signatures of thermal convection, if the temperature dependence of the material parameters is taken into account, so called non-Oberbeck-Boussinesq (NOB) effects. This turns out to mainly be a performance of the boundary layers, as was first shown for water and glycerol (cf. Ahlers 2006, Sugiyama 2007). Another interesting example is the RB convection near the critical point of ethane, which gives some surprising insight into the mechanism leading to NOB effects originating in the strong temperature dependence of the thermal expansion coefficient  $\beta(T)$  and/or the heat capacity  $c_p(T)$ , which mediate the thermal driving of heat convection (cf. Ahlers 2007, 2008).

#### References

Oberbeck A 1879 *Ann Phys Chem* **7** 271

Boussinesq J 1903 *Theorie analytique de la chaleur* **Vol 2** (Gauthier-Villars: Paris)

Ahlers G, Brown E, Fontenele Araujo F, Funfschilling D, Grossmann S and Lohse D 2006 *J Fluid Mech* **569** 409

Sugiyama K, Calzavarini E, Grossmann S and Lohse D 2007 *Eur Phys Lett* **80** 34002

Ahlers G, Fontenele Araujo F, Funfschilling D, Grossmann S and Lohse D 2007 *Phys Rev Lett* **98** 054501

Ahlers G, Calzavarini E, Fontenele Araujo F, Funfschilling D, Grossmann S, Lohse D and Sugiyama K 2008 *Phys Rev E* **77** 046302

---

# Convection beyond the Oberbeck-Boussinesq simplification

## III

### Driving by shear

Siegfried Grossmann et al.

*Fachbereich Physik, Philipps-Universität Marburg, Renthof 6, D-35032 Marburg, Germany*

This third lecture is devoted to recent promising results to explain the momentum (instead of heat) transport in shear driven flows, making use of similar, successful ideas as have been presented in the first two lectures for the heat transport in thermal convection. The relevant systems are Taylor-Couette flow between independently rotating concentric cylinders and turbulent flow through pipes, having angular momentum or axial momentum transport, respectively. At first also for the momentum transport an introduction into the basic physics of shear driven flows based on the Navier-Stokes equation is offered, then the relevant global relations are derived, and finally comparison between theory and the data is provided, which seems quite promising. It turns out that these shear driven flows are non-Oberbeck-Boussinesq-like from the very beginning, although viscosity  $\nu$  is a constant through the liquid here. In contrast, in the shear driven Taylor-Couette system the NOB-like effects originate from the radius dependence of the driving centrifugal force which is relevant here instead of the temperature dependence of the material parameters in thermal flow.

The series of lectures is closed by drawing attention to some most urgent but unsolved open questions in the field, namely plume formation and time dependent boundary layer separation in RB thermal convection, taking "non-Oberbeck-Boussinesqness" into better account for the torque-scaling in Taylor-Couette flow, and extending to the large gap case of TC flow.

#### References

- Esser A and Grossmann S 1996 *Physics of Fluids* **8** 1814  
Eckhardt B, Grossmann S and Lohse D 2000 *Eur Phys J B* **18** 541  
Eckhardt B, Grossmann S and Lohse D 2007 *J Fluid Mech* **581** 221  
Eckhardt B, Grossmann S and Lohse D 2007 *Eur Phys Lett* **78** 24001

# Ergodicity and orbit bunching: Universal features of chaos and their quantum signatures

Alexander Altland, Petr Braun, Fritz Haake, Stefan Heusler, Sebastian Müller

*Transregio-Sonderforschungsbereich "Symmetries and Universality in Mesoscopic Systems", Köln/Bochum/  
Duisburg-Essen/München/Warschau*

Long periodic orbits of fully chaotic dynamics tend to fill the energy shell with uniform density. As a quantitative expression of that well-known fact the sum rule of Hannay and Ozorio de Almeida will be discussed.

More recently, it was discovered that long periodic orbits in chaotic systems do not arise as mutually independent individuals but rather in closely packed bunches. Bunches owe their existence to the fact that each long orbit displays self-encounters in configuration space where two or more orbit stretches run close together. Along each orbit, encounter stretches alternate with "links". Different orbits in a bunch are nearly identical in the links; these links are differently connected by the encounter stretches. I shall reveal this bunching phenomenon as due to a certain exponential stability of the Hamiltonian boundary-value problem which in turn is equivalent to the exponential instability of the initial-value problem of classical mechanics.

Quantum signatures of bunches arise in the semiclassical limit, for all quantities that can be written as sums of Feynman amplitudes of orbits. When a bunch is so closely packed that the orbit-to-orbit action differences become of the order of Planck's constant the Feynman amplitudes interfere constructively. In this way, universal fluctuations in energy spectra or of transport through mesoscopic conductors arise.

Random-matrix theory (RMT) provides a phenomenological description of universal spectral fluctuations, by replacing individual Hamiltonians with random matrices and calculating useful indicators (spacing distribution of energy levels, spectral form factor, . . .) as averages over suitable ensembles of random matrices. Such averages often give simple analytic results. I shall illustrate the fidelity of chaotic dynamics to RMT with a few examples.

## References

- Hannay J H and Ozorio de Almeida A M 1984, *J. Phys. A: Math. Gen.* **17** 3429  
Bohigas O, Giannoni M.-J. and Schmit C 1984, *Phys. Rev. Lett.* **25** 1  
Beenacker C W J 1997, *Rev. Mod. Phys.* **69** 731  
Haake F 2001, *Quantum Signatures of Chaos* (Berlin, Heidelberg, New York: Springer)

# Semiclassical theory of universal spectral fluctuations for chaotic dynamics

Alexander Altland, Petr Braun, Fritz Haake, Stefan Heusler, Sebastian Müller

*Transregio-Sonderforschungsbereich "Symmetries and Universality in Mesoscopic Systems", Köln/Bochum/Duisburg-Essen/München/Warschau*

Gutzwiller's periodic-orbit theory can be used to semiclassically represent the density of energy levels as a formal sum of contributions of periodic orbits. Similarly, the spectral determinant can be approximated as a sum over sets of periodic orbits, the so-called pseudo-orbits. All these sums are divergent, due to the exponential proliferation of periodic orbits with growing period. Only certain correlation functions of such sums can exist and actually be calculated.

This lecture will outline how the two-point correlator of the level density has recently been determined successfully for individual chaotic dynamics, in agreement with the ensemble averages of random-matrix theory. The correlator results as a sum of a non-oscillatory and an oscillatory term; for the Fourier transform with respect to the energy, the "spectral form factor", the two terms mentioned respectively yield the behavior for times up to the Heisenberg time ( $0 < t \leq T_H$ ) and larger ( $T_H < t$ ). The two "terms" of the correlator actually arise as asymptotic series in inverse powers of an energy offset, up to an oscillating exponential factor for the oscillatory term.

For the non-oscillatory part of the two-point correlator one may start from the product of two Gutzwiller sums for the level density. The appropriate starting point for capturing the oscillatory part is a certain generating function, a multiplicative combination of four spectral determinants or their pseudo-orbit representations.

Berry's diagonal approximation involves pairs of identical orbits and gives the leading terms of the asymptotic series involved. Higher-order corrections involve bunches of orbits that differ only in reconnections within close self-encounters.

The contributing orbit bunches look like Feynman diagrams, if close self-encounters are associated with vertices and the intervening links with propagator lines. The resemblance is not at all fortuitous. If the ensemble averages of RMT are implemented perturbatively, within the so-called nonlinear sigma model, Feynman diagrams do uniquely characterize the resulting perturbation series. Each orbit bunch contributing in the semiclassical theory thus turns out one-to-one with a Feynman diagram of RMT.

## References

- Gutzwiller M C 1990, *Chaos in Classical and Quantum Mechanics* (Berlin, Heidelberg, New York: Springer)  
 Berry M V 1985, *Proc. R. Soc. A* **400** 229  
 Sieber M and Richter K 2001, *Phys. Scr.* **T 90** 128  
 Müller S, Heusler S, Braun P, Haake F, and Altland A 2004 *Phys. Rev. Lett.* **93**, 014103 and 2005, *Phys. Rev. E* **72** 046207  
 Heusler S, Müller S, Altland A, Braun P, and Haake F *Phys. Rev. Lett.* **98**, 044103

# Quantum measurement without Schrödinger cat states

Dominique Spehner and Fritz Haake

*Transregio-Sonderforschungsbereich "Symmetries and Universality in Mesoscopic Systems", Köln/Bochum/  
Duisburg-Essen/München/Warschau*

A quantum measurement involves an object  $\mathcal{S}$  and an apparatus; the latter may be idealized to a single-freedom pointer  $\mathcal{P}$  interacting with a many-body bath  $\mathcal{B}$ . The interactions within the three-partite system  $\mathcal{S} \oplus \mathcal{P} \oplus \mathcal{B}$  must (i) entangle  $\mathcal{S}$  and  $\mathcal{P}$  (associating different eigenvalues of the measured object observable with macroscopically distinct pointer displacements) and (ii) decohere the macroscopically distinct pointer states through the action of  $\mathcal{B}$ .

A class of models will be presented which display the behavior just outlined. In particular, emergence and decoherence of distinct pointer displacements can be allowed to proceed simultaneously such that a mixture of macroscopically distinct states arises directly, without any intermediate macroscopic superposition.

Special models involving harmonic oscillators allow for rigorous solutions of the Schrödinger equation of  $\mathcal{S} \oplus \mathcal{P} \oplus \mathcal{B}$ . A much wider model class is amenable to explicit solution in the limit where the time scales for emergence and decoherence of macroscopically distinct states are small compared to the characteristic times of the free motions of  $\mathcal{S}$ ,  $\mathcal{P}$  or even  $\mathcal{S}$ ,  $\mathcal{P}$ ,  $\mathcal{B}$ .

## References

Spehner D and Haake F 2008 *J. Phys. A: Math. Theor.* **41** 072002 and *Phys. Rev.* **E 77** 052114

# Wave Chaos in Rotating Optical Microcavities

Takahisa Harayama

*Department of Nonlinear Science, ATR Wave Engineering Laboratories,  
Kyoto, Japan*

The Sagnac effect is the phase difference between two counter-propagating laser beams in rotating resonators, originally introduced by Sagnac in 1913. It has become the basis for the operation of the optical gyroscopes such as ring laser gyroscopes and fiber optic gyroscopes. These optical gyroscopes are normally used in airplanes, rockets, and ships etc. since they are the most precise rotation velocity sensors among any other types of gyroscopes.

The Sagnac effect had been theoretically derived for the slender waveguides like optical fibers or the ring cavities composed of more than three mirrors by assuming that the light propagates one-dimensionally and the wavelength of the light is much shorter than the sizes of the cavities. However, the sizes of the resonant cavities can be reduced to the order of the wavelength by modern semiconductor technologies. The conventional description of the Sagnac effect is not applicable to such small resonant microcavities. Especially, the resonance wave functions are standing waves which can never be represented by the superposition of counter-propagating waves, while the assumptions of the existence of CW and CCW waves plays the most important role for the conventional theory of the Sagnac effect.

By using perturbation theory typically used in quantum mechanics, we show that the Sagnac effect can also be observed even in resonant microcavities if the angular velocity of the cavity is larger than a certain threshold where the standing wave resonance function changes into the rotating wave. For a quadrupole cavity, it is not assumed that the CW and CCW waves exist in the cavity, but the pair of the counter-propagating waves is automatically produced by mixing the nearly degenerate resonance wave functions due to rotation of the cavity.

We also show that the degenerate eigen-frequency corresponding to the wave-chaotic cavity-mode of the non-rotating cavity splits into two frequencies and their difference is proportional to the rotation rate although the splitting cavity-modes are still wave-chaotic and do not have any corresponding CW and CCW propagating modes as well as ray-dynamical counterparts, which cannot be explained by the conventional Sagnac effect.

## References

- Post E J 1967 *Rev. Mod. Phys.* **39** 475  
Chow W W et al 1985 *Rev. Mod. Phys.* **57** 61  
Sunada S and Harayama T 2006 *Phys. Rev. A* **74** 021801  
Harayama T and Sunada S 2007 *Phys. Rev. E* **76** 016212

# Quantum Dynamical Semigroup Generators

Hiroshi Hasegawa

*Institute of Quantum Science, College of Science and Technology  
Nihon University, Tokyo 101-8308*

We discuss a construction of *quantum dynamical semigroup*, and show (a) derivation of its *generator* from *operator stochastic differential equation*, (b) relation between *dissipation* and *contraction* of the semigroup and (c) *quantum detailed balance*.

## 1. Quantum stochastic differential equation

This is a quantum (i.e. noncommutative) analogue of ordinary stochastic differential equation which was initiated by K.Itô[2], and can be formulated by using the so-called *stochastic calculus* in parallel with the commutative framework.

$$Y \circ dX = Y \cdot dX + \frac{1}{2} dYDX \quad \circ dX \cdot Y + \frac{1}{2} dXdY \quad (1)$$

where the former and the latter expressions must be distinguished for the reason of noncommutativity  $Y \circ dX \neq \circ(dX)Y$ . Also,

$$Z \circ (Y \circ dX) = ZY \circ dX \quad (2)$$

and

$$df(X) = \frac{df}{dX} \circ dX = \frac{df}{dX} dX + \frac{1}{2} d^2 f(X). \quad (3)$$

The symbol  $\circ$  was introduced by Itô [2] for distinguishing between the two Fokker-Planck equations of Itô and Stratonovich(see [1]).

## 2. Stochastic Schroedinger and stochastic Heisenberg equation

We show a more concrete incorporation of the stochastic element into the ordinary quantum dynamics: our purpose is to elucidate how statistical feature *dissipativeness* can be formulated on the basis of three rules summarized by eq.(1), eq.(2) and eq.(3). The result can be shown in terms of *quantum dynamical semigroup* and its generator as follows.

$$X_t = \Lambda_t X \equiv e^{tL} X_{t=0} \quad (4)$$

where  $\Lambda_t$  is a *superoperator* acting on the space of operators for a given quantum system.

In the above, the superoperator  $L$  in  $\Lambda_t$  is called *generator* of the (quantum) dynamical semigroup.

## 3. Condition of detailed balance

Let us recall the classical description of *detailed balance condition*[9] expressed as

$$\Sigma_l \frac{\partial a_{il}^{-1} b_l}{\partial x_j} = \Sigma_l \partial a_{jl}^{-1} b_l \partial x_i, \quad (5)$$

where

$$\frac{\partial p(t, x)}{\partial t} - \frac{\partial}{\partial x_i} (-b_i(x)p(t, x)) + \frac{1}{2} \frac{\partial}{\partial x_j} (a_{ij}(x) \frac{\partial}{\partial x_i} a_{ij}(x)p(t, x)) \quad (6)$$

in terms of *drift vector*  $b_i(x)$  and *diffusion tensor* element  $a_{ij}(x)(= a_{ji}(x))$ . Then, the detailed balance condition is given by eq.(5) [9].

A possible quantum mechanical detailed balance condition can be formulated based on (i) existence of the equilibrium density operator  $\omega$  which is unique (ii) the set of unitary time evolutions  $\Sigma_t \equiv \{U_t; -\infty < t < \infty\}$  commutes with the semigroup generator  $L$  defined in eq.(4).

The details of the above feature will be presented.



---

## References

- [1] H Hasegawa and T Monnai 2005 *Entropy Production in Nonequilibrium Systems*, unpublished
- [2] K Itô 1975 *Lecture Notes in Physics* **39** 218
- [3] R Bhatia 1907 *Matrix Analysis* (Springer Verlag, Berlin)
- [4] R L Hudson and K R Parthasarathy 1984 *Commun. Math. Phys.* **93** 301
- [5] G Lindblad 1976 *Commun. Math. Phys.* **48** 119
- [6a] K Kraus 1970 *Ann. Phys.* **64** 311;
- [6b] Størmer 1974, *Lecture Notes in Phys.* **29** 85
- [7] D Petz 2002, *J. Phys. A: Math. Gen.* **35** 929
- [8] R L Stratonovich 1968, *Conditional Markov processes and their applications to optimal control* (Elsevier, New York)
- [9] R Graham and H Haken 1971, *Z. Physik* **243** 283
- [10] R Alicki 1976, *Rep. Math. Phys.* **10** 249
- [11] H Hasegawa and T Nakagomi, *J. Stat. Phys.* **23** 639

# Asymmetric Heat Conduction in Nonlinear Systems

**Bambi Hu**

*Department of Physics, Hong Kong Baptist University,  
Kowloon Tong, Hong Kong, China*

*and*

*Department of Physics, University of Houston, Houston,  
TX 77204-5005, USA*

Heat conduction is an old yet important problem. Since Fourier introduced the law bearing his name two hundred years ago, a first-principle derivation of this law from statistical mechanics is still lacking. Worse still, the validity of this law in low dimensions, and the necessary and sufficient conditions for its validity are still far from clear. In this talk I'll give a review of recent works done on this subject. I'll also report our latest work on asymmetric heat conduction in nonlinear systems. The study of heat conduction is not only of theoretical interest but also of practical interest. The study of electric conduction has led to the invention of such important electric devices such as electric diodes and transistors. The study of heat conduction may also lead to the invention of thermal diodes and transistors in the future.

---

# Semiclassical assignment of highly excited molecular vibrations

Christof Jung

*Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México  
Cuernavaca, México*

We present a method to treat highly excited molecular vibrations of small polyatomic molecules. We start from an algebraic Hamiltonian either fitted to the experimental spectrum or constructed from a given potential surface. Its classical counterpart  $H$  has several very favorable properties: It has a natural decomposition into an integrable  $H_0$  and resonant interactions  $W$ . The whole Hamiltonian is automatically given in the action/angle variables belonging to  $H_0$ . The configuration space of the system is the angle space. Polyad type conserved quantities allow a reduction of the number of degrees of freedom.

If some of the modes are involved in several independent resonances, then the system is nonintegrable and in many cases shows chaos in large parts of the classical phase space.

First we study the various coupling schemes possible by the functional form of  $W$  and the corresponding organization centers in configuration space. Then we construct a semiclassical representation on the configuration space of the eigenstates of the quantum problem and compare them with the organization centers of the various coupling schemes. For the large majority of states we thereby see to which coupling scheme they belong. By counting nodal patterns and phase advances of the semiclassical wave functions we get quantum numbers relative to the respective organization center. As principal example DCO is used. Other examples are mentioned briefly.

The success of this method for systems with a complicated spectrum and with classical chaos poses interesting questions on quantum chaology. Finally we make some suggestions for a possible answer of this question.

# Spatial network representation of complex living tissues

**Dean Korošak<sup>1</sup> and Marjan Rupnik<sup>2</sup>**

<sup>1</sup>*CAMTP - Center for Applied Mathematics and Theoretical Physics,  
University of Maribor, Maribor, Slovenia*

<sup>2</sup>*Faculty of Medicine,  
University of Maribor, Maribor, Slovenia*

Complex network theory has recently attracted large attention across several scientific disciplines as it can be used to describe very diverse systems from the network perspective. In this approach the elements of the system are represented by vertices and interactions between the elements by edges or links connecting the vertices. We shall first review the basic elements of network theory focusing on the scale-free networks and spatially embedded scale-free networks in particular. We shall introduce and discuss some of the algorithms for constructing networks with scale-free architecture.

Following this, we will present a network approach to explore the relationship between structure and function in biological systems, concentrating on the case of pancreatic islets of Langerhans. We will describe a structure of an intact living islet of beta cells as a scale-free complex system using a fitness network model embedded in Euclidean space. The connectivity between two beta cells in the model is the function of the spatial localization and conductance between these cells. Both quantities have been experimentally determined in intact pancreatic islets. Several computed properties of the networks representing the intact pancreatic islets will be presented such as degree distributions, degree correlations and centrality. We shall argue that by analyzing spatial network models of pancreatic islets we can progress our understanding of the interrelation between the structural code and functional connections inside living pancreatic islets.

## References

- Albert R and Barabasi AL 2002 *Rev. Mod. Phys.* **74** 47.  
Barabasi A-L and Albert R 1999 *Science* **286** 509.  
Cabrera O, Berman DM, Kenyon NS, Ricordi C, Berggren P-O and Caicedo A 2006 *PNAS* **103** 2334.  
Caldarelli G, Capocci A, De Los Rios P, and Munoz MA 2002 *Phys. Rev. Lett.* **89** 258702.  
Speier S, Rupnik M 2003 *Pflügers Arch* **446** 553.

---

# Magnetic domain patterns under an oscillating field

**Kazue Kudo and Katsuhiko Nakamura<sup>1,2</sup>**

*Ochadai Academic Production, Ochanomizu University, Tokyo, Japan*

<sup>1</sup>*Department of Applied Physics, Osaka City University, Osaka, Japan*

<sup>2</sup>*Heat Physics Department, Uzbek Academy of Sciences, Tashkent, Uzbekistan*

Magnetic domain patterns show various kinds of structures under a magnetic field. Especially under an oscillating field, one can observe interesting domain patterns such as square/hexagonal lattices, travelling stripe patterns, spirals, concentric circles, etc.

We discuss domain patterns in a ferromagnetic thin film under an oscillating field. The domain patterns consist of up-spin and down-spin clusters. We consider domain patterns with a constant characteristic width. The characteristic width is determined by the balance between the exchange interactions (short-range attractive interactions) and the dipolar interactions (long-range repulsive interactions). We use a simple two-dimensional Ising-like model including those interactions in order to simulate various kinds of domain patterns and discuss the characteristics and mechanism of the patterns.

Under some conditions, lattice patterns are observed in both experiments and numerical simulations. We show numerical simulations of some kinds of lattice patterns. The type of a lattice (i.e. square or hexagonal) depends on the amplitude and frequency of an oscillating field. We also show the numerical simulation of a travelling stripe pattern. In fact, travelling patterns can be observed in experiments. We discuss the mechanism and instability of a travelling pattern. Furthermore, we will also discuss more interesting patterns like spirals and concentric circles.

## References

Cross M C and Hohenberg P C 1993 *Rev. Mod. Phys.* **65** 851

Seul M and Andelman D 1995 *Science* **267** 476

Tsukamoto N, Fujisaka H and Ouchi K 2006 *Prog. Theor. Phys. Suppl.* **161** 372

Kudo K, Mino M and Nakamura K 2007 *J. Phys. Soc. Jpn* **76** 013002

Kudo K and Nakamura K 2007 *Phys. Rev. E* **76** 036201

# Spatio-temporal modelling of intracellular $\text{Ca}^{2+}$ oscillations

Marko Marhl

*Department of Physics, Faculty of Natural Sciences and Mathematics,  
University of Maribor, Maribor, Slovenia*

In many non-excitable eukaryotic cells  $\text{Ca}^{2+}$  oscillations play a key role in intra and intercellular signalling, thus regulating many cellular processes from fertilization to death (Berridge et al., 1998). Mathematical modelling of  $\text{Ca}^{2+}$  oscillations is of particular importance for understanding the mechanisms underlying these oscillations, and consequently understanding how they may be regulated. The experiments in hepatocytes show that when net plasma membrane  $\text{Ca}^{2+}$  efflux is reduced, and hence the total concentration of  $\text{Ca}^{2+}$  in the cell increases, the frequency of  $\text{Ca}^{2+}$  oscillations increases as well, but importantly, the width of the spikes remains constant (Green et al., 1997, 2002, 2003). The existing mathematical models (for review see Schuster et al., 2002) were not able to explain this phenomenon. We show that these experimental observations can be best explained by taking into account not only the temporal, but also the spatial dynamics underlying the generation of  $\text{Ca}^{2+}$  oscillations in the cell (Marhl et al., 2008). Here we divide the cell into a grid of elements and treat the  $\text{Ca}^{2+}$  dynamics as a spatio-temporal phenomenon. Thus converting an existing temporal model into a spatio-temporal one delivers results that are in much better agreement with experimental observations.

## References

- Berridge, M.J., Bootman, M.D., Lipp, P. 1998. Calcium - a life and death signal. *Nature (London)* **395**, 645-648.
- Green, A.K., Cobbold, P.H., Dixon, C.J., 1997. Effects on the hepatocyte  $[\text{Ca}^{2+}]_i$  oscillator of inhibition of the plasma membrane  $\text{Ca}^{2+}$  pump by carboxyeosin or glucagon-(19-29). *Cell Calcium* **22**, 99-109.
- Green, A. K., Zolle, O., Simpson, A.W.M., 2002. Atrial Natriuretic Peptide Attenuates  $\text{Ca}^{2+}$  Oscillations and Modulates Plasma Membrane  $\text{Ca}^{2+}$  Fluxes in Rat Hepatocytes. *Gastroenterology* **123**, 1291-1303.
- Green, A.K., Zolle, O., Simpson, A.W.M., 2003. Regulation of  $[\text{Ca}^{2+}]_i$  oscillations by plasma membrane  $\text{Ca}^{2+}$  fluxes: a role for natriuretic peptides. *Biochem. Soc. Trans.* **31**, 934-938.
- Marhl, M., Gosak, M., Perc, M., Green, A.K., Dixon, C.J., 2008. Spatio-temporal modelling explains the effect of reduced plasma membrane  $\text{Ca}^{2+}$  efflux on intracellular  $\text{Ca}^{2+}$  oscillations in hepatocytes. *J. Theor. Biol.* **252**, 419-426.
- Schuster, S., Marhl, M., Höfer, T., 2002. Modelling of simple and complex calcium oscillations. From single-cell responses to intercellular signalling. *Eur. J. Biochem.* **269**, 1333-1355.

# Wave turbulence in superfluid $^4\text{He}$ : energy cascades and rogue waves in the laboratory

A. N. Ganshin,<sup>1</sup> V. B. Efimov,<sup>1,2</sup> G. V. Kolmakov,<sup>2,1</sup> L. P. Mezhov-Deglin,<sup>2</sup> and P. V. E. McClintock<sup>1</sup>

<sup>1</sup>*Department of Physics, Lancaster University, Lancaster LA1 4YB, UK*

<sup>2</sup>*Institute of Solid State Physics RAS, Chernogolovka, Moscow 142432, Russia*

A highly excited state of a system with numerous degrees of freedom, characterized by a directional energy flux through frequency scales, is referred to as *turbulent*. Like the familiar manifestations of vortex turbulence in fluids, turbulence can also occur in systems of waves, e.g. turbulence of sound waves in oceanic waveguides, magnetic turbulence in interstellar gases, shock waves in the solar wind and their coupling with Earth's magnetosphere, and phonon turbulence in solids. Following the ideas of Kolmogorov, the universally accepted picture says that nonlinear wave interactions give rise to a cascade of wave energy towards shorter and shorter wavelengths until, eventually, it becomes possible for viscosity to dissipate the energy as heat. Experiments and calculations show that, most of the time, the Kolmogorov picture is correct. We have found, however, that this picture is incomplete. Our experiments with second sound waves in superfluid  $^4\text{He}$  show that, contrary to the conventional wisdom, acoustic wave energy can sometimes flow in the opposite direction.

We shall review briefly the necessary background in turbulence and superfluidity, discussing why superfluid  $^4\text{He}$  is an ideal medium for modelling wave turbulence in the laboratory. Then we will report recent results and discuss their implications in physics and in relation to possible applications such as rogue waves on the ocean.

## References

- Dyachenko A I and Zakharov V E 2005 *JETP Lett.* **81**, 255 (2005)  
 Gurbatov S N, Kurin V V, Kustov L M and Pronchatov-Rubtsov N V 2005 *Acoust. Phys.* **51** 152  
 Kolmakov G V, Levchenko A A, Brazhnikov M Yu, Mezhov-Deglin L P, Silchenko A N and McClintock P V E 2004 *Phys. Rev. Lett.* **93** 074501  
 Kolmakov G V, Efimov V B, Ganshin A N, McClintock P V E and Mezhov-Deglin L P 2006 *Phys. Rev. Lett.* **97** 155301  
 Kolmogorov, A N 1941 *Dokl. Akad. Nauk. SSSR* **30**, 299  
 Nazarenko S 2007 *JETP Lett* **84** 585  
 Vinen W F, Tsubota M and Mitani A 2003 *Phys. Rev. Lett.* **86** 135301  
 Zakharov V E, Falkovich G and L'vov V S 1992 *Kolmogorov Spectra of Turbulence I* (Springer, Berlin, 1992)

# Clustering of inertial particles in turbulent flows

B. Mehlig<sup>1)</sup> and M. Wilkinson<sup>2)</sup>

<sup>1)</sup>*Department of Physics, Göteborgs universitet, Gothenburg, Sweden*

<sup>2)</sup>*The Open University, Milton Keynes, England*

We consider particles suspended in a randomly stirred or turbulent fluid. When effects of the inertia of the particles are significant, an initially uniform scatter of particles can cluster together. We analyse this ‘unmixing’ effect by calculating the Lyapunov exponents for dense particles suspended in such a random three-dimensional flow, concentrating on the limit where the viscous damping rate is small compared to the inverse correlation time of the random flow (that is, the regime of large Stokes number). In this limit Lyapunov exponents are obtained as a power series in a parameter which is a dimensionless measure of the inertia. We report results for the first seven orders. The perturbation series is divergent, but we obtain accurate results from a Padé-Borel summation. We infer that particles can cluster onto a fractal set and show that its dimension is in satisfactory agreement with previously reported results from simulations of turbulent Navier-Stokes flows [Bec *et al.* 2006].

## References

- Balkovsky E, Falkovich G, and Fouxon A 2001 *Phys. Rev. Lett.* **86** 2790  
Bec J, Biferale L, Boffetta G, Cencini M, Musachchio S, and Toschi F 2006 *Phys. Fluids* **18** 091702  
Duncan K, Mehlig B, Östlund S, and Wilkinson M 2005 *Phys. Rev. Lett.* **95** 240602  
Kaplan J L, and Yorke J A 1979, in: *Functional Differential Equations and Approximations of Fixed Points*, Lecture Notes in Mathematics, edited by H.-O. Peitgen and H.-O. Walter, Springer, Berlin, Vol. 730, p. 204  
Le Jan Y 1985 *Z. Wahrscheinlichkeitstheor. Verwandte Geb.* **70** 609  
Maxey M 1987 *J. Fluid Mech.* **174** 441  
Maxey M, and Riley J 1983 *Phys. Fluids* **26** 883  
Mehlig B, and Wilkinson M 2004 *Phys. Rev. Lett.* **92** 250602  
Sommerer J, and Ott E 1993 *Science* **359** 334  
Wilkinson M, Mehlig B, Östlund S, and Duncan K 2007 *Phys. Fluids* **19** 113303



---

# Relative speeds of inertial particles at large Stokes numbers

**B. Mehlig<sup>1)</sup> and M. Wilkinson<sup>2)</sup>**

<sup>1)</sup>*Department of Physics, Göteborgs universitet, Gothenburg, Sweden*

<sup>2)</sup>*The Open University, Milton Keynes, England*

We discuss the probability distribution of relative speed  $\Delta V$  of inertial particles suspended in a highly turbulent gas when the Stokes numbers, a dimensionless measure of their inertia, is large. We identify a mechanism giving rise to the distribution  $P(\Delta V) \sim \exp(-C|\Delta V|^{4/3})$  (for some constant  $C$ ). Our conclusions are supported by numerical simulations and the analytical solution of a model equation of motion. The results determine the rate of collisions between suspended particles.

## References

Gustavsson K, Mehlig B, Wilkinson M, and Uski V 2008 [arXiv:0802.2710](https://arxiv.org/abs/0802.2710)

Mehlig B, Uski V, and Wilkinson M 2007 *Phys. Fluids* **19** 098107

Völk H J, Jones F C, Morfill G E, Röser S 1980 *Ann. N.Y. Acad. Sci.* **85** 316

# Synchronization in large networks of coupled phase oscillators: the effect of network topology

Edward Ott

*Institute for Research in Electronics and Applied Physics,  
University of Maryland, College Park, Maryland 20742*

Synchronization of large systems of coupled oscillators is a basic issue in settings ranging from brain function, to electrical circuits, to laser arrays, etc. This lecture will begin with a review of the paradigmatic Kuramoto model of globally coupled phase oscillators where each oscillator has a different intrinsic frequency drawn from some prescribed probability distribution function [Ott 2002]. We then introduce concepts of network connectivity and formulate the problem of phase oscillators coupled on a network. The problem is analyzed in three stages of approximation. It is found that the effect of network topology on synchronization is mainly characterized by the largest eigenvalue  $\lambda$  of the network adjacency matrix. Based on this, one can assess the influence upon synchronizability of such network attributes as diversity in the number of connections to network nodes, spread in the link strengths, and the tendency for highly connected nodes to connect to other highly connected nodes (assortativity).

## References

Section 6.6 of the book *Chaos in Dynamical Systems* by E. Ott (Cambridge Univ. Press, 2002) contains a review of the Kuramoto model.

Restrepo J G, Ott E and Hunt B R 2005 *Phys. Rev. E* **71** 036151

Restrepo J G, Ott E and Hunt B R 2006 *Chaos* **16** 015107

Restrepo J G, Ott E and Hunt B R 2007 *Phys. Rev. E* **76** 056119

---

# Emergence of collective behavior in large networks of coupled heterogeneous dynamical systems

Edward Ott

*Institute for Research in Electronics and Applied Physics,  
University of Maryland, College Park, Maryland 20742*

After a review and introduction of the subject, this lecture will be devoted to investigations of large systems of coupled heterogeneous dynamical systems. The first part of the lecture will deal with how interactions with external drivers influence the dynamics. Examples in this category include the interaction of the walking dynamics of pedestrians on London's Millennium Bridge that lead to unexpected violent oscillations of the bridge [Strogatz et al 2005, Eckhardt et al 2007] circadian rhythm governing the sleep-awake cycle of animals through the coupling of many oscillatory neurons in the brain's superchiasmatic nucleus influenced by the daily 24 hour variation of sunlight [Antonsen et al], etc. The second part of the talk will be devoted to the study of systems of many coupled heterogeneous dynamical systems, including cases where the coupled systems can be chaotic. As the coupling strength is increased, a transition from incoherent to coherent collective behavior is typically observed. A general theory for this transition will be presented [Ott et al 2002, and Baek and Ott 2004]. An important point is that the macroscopic (i.e., averaged) system behavior is typically periodic even if the systems that are coupled behave chaotically. We present a treatment for the case of global coupling and then indicate how it can be generalized to situations with nontrivial network topology [Restrepo et al 2006].

## References

- Strogatz S H, Abrams D, McRobie A, Eckhardt B and Ott E 2005 *Nature* **438** (7048), 43  
Eckhardt B, Ott E, Strogatz S H, Abrams D and McRobie A 2007 *Phys. Rev. E* **75** 021110  
Antonsen T M, Faghih R, Girvan M, Ott E and Platig J, arXiv:0711.4135  
Ott E, So P, Barreto E and Antonsen T M 2002 *Physica D* **173** 29  
Baek S -J and Ott E 2004 *Phys. Rev. E* **69** 066201  
Restrepo J G, Ott E and Hunt B R 2006 *Phys. Rev. Lett.* **96** 254103; *Physica D* **224** 114

# Estimating the state of large spatiotemporally chaotic systems, weather forecasting, etc.

Edward Ott

*Institute for Research in Electronics and Applied Physics,  
University of Maryland, College Park, Maryland 20742*

State estimation is a general requirement for controlling a system or for predicting its future evolution., In this talk we will address the problem of estimating the state of a large spatiotemporally chaotic system from limited noisy measurement data and a knowledge of a system model. For large systems, state estimation can be particularly challenging because straightforward application of the conventional techniques is typically not feasible due to computational limitations. This problem has very general interest, e.g., for weather forecasting, etc. This talk will present background material, a proposed solution for treating large systems, and illustrative results from application of our technique to weather forecasting and to a laboratory experiment.

## References

Ott E et al 2004 *Tellus A* **56** 415; *Phys. Lett. A* **330** 365  
Szunyogh I et al 2005 *Tellus A* **57** 528

# The Limits of Some Infinite Families of Complex Contracting Mappings

Dušan Pagon

*Faculty of Natural Science and Mathematics,  
University of Maribor, Maribor, Slovenia*

The idea of self-similarity is one of the most fundamental in modern mathematics. The notion of renormalization group, which plays an essential role in quantum field theory, statistical physics and dynamical systems, is related to it. Many fractal and multi-fractal objects, playing an important role in singular geometry, measure theory and holomorphic dynamics, are related. Self-similarity also appears in the theory of  $C^*$ -algebras (for example in the representation theory of the Cuntz algebras) and in many other branches of mathematics. For the last 25 years the idea of self-similarity demonstrates a growing influence on asymptotic and geometric group theory.

A compact set  $F$  in a metric space is called self-similar (in a strong sense) with a similarity coefficient  $p$ , when it can be divided into  $N$  congruent sets, each of which is exactly  $p$ -times smaller copy of the original set. Our main purpose is a new approach to a broad class of planar fractal sets. We discuss some strongly selfsimilar sets of points in the complex plane, obtained as geometric limits of certain infinite families of contracting mappings (homotheties with  $q = \frac{1}{p} < 1$ ). Some well known fractal sets (like the Sierpinski gasket) are included in this class, but generally new planar fractal sets appear, that have not been studied before.

For an arbitrary stretching factor  $r \in (0, 1)$  and a positive angle  $\vartheta < \pi$  we define an infinite family of bijective affine mappings of the complex plane, consisting of a rotation of the plane (for a certain multiple of the angle  $\vartheta$ ) with the appropriate stretching (for the related power of factor  $r$ ) and some translation. Starting with a unit segment we repeatedly use this iterated functions system on it. The limiting set of points is under certain conditions in a 1–1 correspondence with the initial unit interval on the real axis.

## References

- Beardon A F 1991 *Iteration of rational functions. Complex analytic dynamical systems, Grad. Texts in Math* **132** (New York: Springer-Verlag).
- Fried D L 1987 *Ergod. Th. Dynam. Sys.* **7** pp489–507.
- Grunbaum B, Shephard G C 1987 *Tilings and patterns* (New York: W. H. Freeman).
- Hata M 1985 *Japan J. Appl. Math.* **2** pp381–414.
- Kitchens B P 1998 *Symbolic dynamics* (Berlin: Springer).
- Pagon D 2003 *Prog. Theor. Phys, Suppl. (Kyoto)* **150** pp176–185.
- Pagon D 2004 *Int. J. of Comp. and Num. Analysis and Appl.* **6** 1 pp65–76.
- Zeitler H, Pagon D 2000 *Fraktale Geometrie* (Braunschweig: Vieweg Verlag).

# Cyclical interactions, defensive alliances, and the Phoenix effect

Matjaž Perc

*Department of Physics, Faculty of Natural Sciences and Mathematics,  
University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia*

Cyclical interactions are simple yet fascinating and powerful examples of evolutionary processes, able to provide insights into the intriguing mechanisms of Darwinian selection [1] as well as structural complexity [2] and pre-biotic evolution [3]. The simplest non-trivial food web describing such cyclical interactions is formed by three species that have relationships analogous to the well-known rock-scissors-paper (RSP) game, where strategies form a closed loop of dominance.

We will first outline some general properties of the RSP game and present Monte Carlo simulations that will help us to get familiar with the basic premise of cyclical interactions. Next, an extension of the basic RSP game, given in the form of a six species model (or food web) will be presented. We will show that the more complex version, supplemented also by heterogeneous invasion rates [4], exhibits many interesting features that can be encountered in the human and animal world. Besides cyclical interactions as the basic ingredient, we will also show and explain the existence of defensive alliances, demonstrate the Phoenix effect, as well as noise-guided evolution [5].

Finally, we will also link the subject with evolutionary game theory [6] and present our recent results showing that static [7] and dynamic [8,9] stochastic inputs may play a decisive role in determining the evolutionary success of participating strategies. Interested individuals are kindly invited to write me an email\* or visit my homepage† to learn more about the subject.

## References

- [1] Maynard Smith J and Price G R 1973 *Nature* **246** 15
- [2] Watt A S 1947 *J. Ecol.* **35** 1
- [3] Rasmussen S et al. 2004 *Science* **303** 963
- [4] Perc M, Szolnoki A and Szabó G 2007 *Phys. Rev. E* **75** 052102
- [5] Perc M and Szolnoki A 2007 *New J. Phys.* **9** 267
- [6] Szabó G and Fáth G 2007 *Phys. Rep.* **446** 97
- [7] Perc M and Szolnoki A 2007 *Phys. Rev. E* **77** 011904
- [8] Perc M 2006 *New J. Phys.* **8** 22
- [9] Perc M 2007 *Econ. Lett.* **97** 58

\*matjaz.perc@uni-mb.si; †<http://matjazperc.com/>

# Virtual Volatility

Richard E. Prange, A. Christian Silva

*Physics Department, University of Maryland, College Park, Md 20742*

A basic investment strategy is: Buy a stock (or portfolio of stocks) at price  $S_0$  at time  $t = 0$  intending to sell it a ‘year’ later at time  $t = T$  at a higher price  $S_T$ . A widely adopted semirealistic model for ‘predicting’ stock prices is the *random walk*. Then the price changes ‘daily’ by a small fraction  $ds_n = (S_{(n+1)dt} - S_{ndt})/S_{ndt} = \mu dt + \sigma\sqrt{dt}W_n$  where  $dt = T/N$ , ( $N \gg 1$ ) and  $W_n$  is random with zero mean and unit variance. The Central Limit Theorem then shows that the *predicted* probability distribution function (PDF) of (*log*) returns  $R_T = \ln(S_T/S_0)$  is approximately normal (Gaussian). The center of the PDF is the ‘*expected*’ return  $\mu_T = N\mu dt$ . The PDF’s width,  $\sigma_T = \sqrt{Ndt}\sigma$ , is a version of the *volatility*, commonly but incorrectly equated to the *risk*. Modern Portfolio Theory, MPT, and the Capital Assets Pricing Model CAPM, still in every textbook, make the ridiculous assumption that every market participant knows and indeed agrees on the ‘true’ values of  $\mu_T$  and  $\sigma_T$  for each and every portfolio of stocks. Because there is just one exemplar, this PDF *cannot be measured*, even when time  $T$  is history. Nevertheless, we agree that the shape is roughly normal. We argue that  $\sigma_T$  is known with maybe 20% accuracy and typically is in the range 10%-100%. In contrast,  $\mu_T$  is only crudely estimated, e.g.  $\mu_T \approx .5\sigma_T \pm \sigma_T$ . This triviality implies what is obvious to us, but not to economists, that the actual uncertainty  $\sigma_{TV}$  of an investor’s *predicted return PDF* is *larger* than the random walk (CAPM or Black-Sholes) width, because of inability to predict well the expected return parameter  $\mu_T$ . We call this corrected width the *virtual volatility* VV.

We estimate the size of the effect empirically and find that  $\sigma_{TV} \approx 1.3\sigma_T$ . We also confirm the normality of the PDF of returns. Remarkably, for a random walk with *fixed* but *unknown* parameters, there is a *bigger* VV effect,  $\sigma_{TV} = 1.45\sigma_T$ . Our result is evidence of a *market ‘anomaly’*, a breakdown of the *efficient market hypothesis*, something not routinely accepted by most academic economists: The simple random walk must be replaced by a *mean reverting random walk*. We extract from the data an empirical *time to revert to the mean* of about 15 months.

The VV also has dramatic consequences on the choice of optimal investment vehicle. While the VV effect obviously reduces the advisability of buying a stock directly, the extra uncertainty actually *improves* the strategy of buying *calls* on the underlying stock. Thus, in this case, *uncertainty* can be a good thing, confirming the adage *ignorance is bliss!*

# Third quantization: a general method to solve master equations for quadratic open Fermi systems

**Tomaž Prosen**

*Faculty of Mathematics and Physics - Department of Physics  
University of Ljubljana, Ljubljana, Slovenia*

In this lecture we shall outline a general approach to explicit solution of quantum Liouville equations for open many-body systems out of equilibrium [1].

Using the concept of quantization in the Fock space of operators, the Lindblad master equation for an arbitrary *quadratic* system of  $n$  fermions shall be solved explicitly in terms of diagonalization of a  $4n \times 4n$  matrix, provided that all Lindblad bath operators are *linear* in the fermionic variables [1].

As an example, the method is applied to the explicit construction of non-equilibrium steady states and the calculation of asymptotic relaxation rates in the far from equilibrium problem of heat and spin transport in a nearest neighbor Heisenberg  $XY$  spin  $1/2$  chain in a transverse magnetic field. Furthermore, we find and demonstrate a novel type of far from equilibrium quantum phase transition with spontaneous emergence of long-range order in spin-spin correlation functions [2].

## References

- [1] Prosen T 2008 *New J. Phys.* **10** 043026
- [2] Prosen T and Pižorn I 2008 preprint [arXiv:0805.2878](https://arxiv.org/abs/0805.2878)



# Quantum chaos in many-body systems

**Tomaž Prosen**

*Faculty of Mathematics and Physics - Department of Physics  
University of Ljubljana, Ljubljana, Slovenia*

Several approaches to general characterizations of non-integrable interacting many-body systems shall be reviewed [1]. We shall focus on quantum spin 1/2 chains as a particular convenient class of generic systems and outline several dynamical signatures of (non)integrability, employing concepts from random matrix theory (see e.g. [2]) and from quantum information theory (see e.g. [3, 4]). In particular we shall discuss (i) energy level statistics and (generalized) quantum chaos conjecture [5], (ii) decay of dynamical correlation functions with equilibrium relaxation rates as quantum analogues of Ruelle resonances [1], and the relation to quantum transport problem in non-equilibrium statistical mechanics [6], (iii) algorithmic complexity, entanglement production and the efficiency of classical simulations of quantum dynamics [7], and (iv) computation of quantum dynamical entropies [1].

These criteria and their ranges of validity will be discussed and compared, and sometimes quite surprising conclusions are found. Some conjectures and open problems in the ergodic theory of the quantum many problem shall be suggested.

## References

- [1] Prosen T 2007 *J. Phys. A: Math. Theor.* **40** 7881
- [2] Haake F 2001 "Quantum Signatures of Chaos", 2nd Edition, (Springer, Heidelberg)
- [3] Benenti G, Casati G and Strini G "Principles of Quantum Computation and Information. Volume I: Basic Concepts" (World Scientific, Singapore 2004)
- [4] Benenti G, Casati G and Strini G "Principles of Quantum Computation and Information. Volume II: Basic Tools and Special Topics" (World Scientific, Singapore 2007)
- [5] Pineda C and Prosen T 2007 *Phys. Rev. E* **76** 061127
- [6] C. Mejia-Monasterio, T. Prosen and G. Casati 2005 *Europhys. Lett.* **72** 520
- [7] Prosen T and Žnidarič M 2007 *Phys. Rev. E* **75** 015202

# Dynamics of Loschmidt echoes and fidelity decay

**Tomaz Prosen**

*Faculty of Mathematics and Physics - Department of Physics  
University of Ljubljana, Ljubljana, Slovenia*

Fidelity serves as a benchmark for the reliability in quantum information processes, and has recently attracted much interest as a measure of the susceptibility of general dynamics to external perturbations. A rich variety of regimes for fidelity decay have emerged. In this lecture we shall review some of the most important regimes, and outline the theory that supports them [1]. While we mention several theoretical approaches we use time correlation functions as a backbone for the discussion. Recent experiments in micro-wave cavities and in elastodynamic systems as well as suggestions for experiments in quantum optics shall be discussed.

## References

[1] Gorin T, Prosen T, Seligman TH and Žnidarič M 2006 *Phys. Rep.* **435** 33

# Topology and chaos

Dušan Repovš

*Institute of Mathematics, Physics, and Mechanics  
University of Ljubljana, P. O. Box. 2964, Ljubljana, Slovenia 1000*

In this lecture we shall illustrate, by means of examples, how certain methods of modern geometric topology can be used to construct very interesting and diverse examples in chaos:

(i) Using a classical example from 1930's, due to J. H. C. Whitehead, of an open contractible 3-manifold which fails to be  $R^3$ , we shall see how the corresponding "continuum at infinity" arises as a chaotic local attractor for a special self-homeomorphism of  $R^3$ .

(ii) Using interval mappings, we shall demonstrate that every inverse limit space of such mappings can be realized as a global attractor for a homeomorphism of  $R^2$ .

(iii) Using Borsuk's shape theory, we shall show how one can get a compactum (a certain one-dimensional space called a "solenoid") which cannot be an attractor of any self-map of a topological manifold.

(iv) Using forcing of periodic points in orientation-reversing twist maps of the plane (for example, the Hénon maps), we shall show that an orientation-reversing twist map can be written as the composition of four orientation-preserving twist maps.

(v) Applying the Conley index to the dynamics of  $f$ , we shall show that period-4 points can be divided into two types and prove that if  $f$  is a diffeomorphism, then the existence of a point of type I implies that there exists a compact invariant subset  $\Lambda \subset R^2$  such that  $f|_{\Lambda}$  has positive topological entropy.

## References

- Barge M., Martin J. 1990 *Proc. Amer. Math. Soc.* **110** 523  
 Cencelj M., Repovš D. 2001 *Topologija* (Ljubljana: Univerza v Ljubljani)  
 Garity D.J., Jubran I.S. and Schori R.M. 1997 *Houston J. Math.* **23** 33  
 Ghrist R.W., van den Berg J.B. and Van der Vorst R.C.A.M. 2003 *Invent. Math.* **152** 369  
 Gilmore R., Lefranc M. 2002 *The Topology of Chaos: Alice in Stretch and Squeezland* (Hoboken: Wiley)  
 Günther B. 1994 *Proc. Amer. Math. Soc.* **120** 653  
 Kennedy J., Yorke J.A. 2001 *Trans. Amer. Math. Soc.* **353** 2513  
 van den Berg J.B., Vandervorst R.C., Wójcik W. 2007 *Topology Appl.* **154** 2580.

# Chaotic motion in rigid body dynamics

Peter H. Richter

*Institute for Theoretical Physics,  
University of Bremen, Bremen, Germany*

The theory of rigid body motion in a gravitational field has a long history. It is intimately connected with the names of Leonard Euler, Joseph Louis de Lagrange, and Sophia Kovalevskaya – each of them identified an integrable family of bodies which now carry their names. The beautiful mathematics of elliptic and hyperelliptic functions was developed in that connection, but little attention was given to the fact that the overwhelming majority of rigid body problems is non-integrable: their typical dynamics is more or less chaotic.

The lecture will outline the nature of the problem and the strategy of its investigation. First we must take into account that realistic rigid bodies need a device to fix a point which is not the center of gravity – a Cardan suspension for example. The configuration space is then no longer  $SO(3)$  where Euler's angles or Euler's elegant variables may be used as coordinates, but a 3-torus of Cardan angles. Two other implications are that (i) the set of all these physical systems has (at least) six essential parameters and (ii) the six-dimensional phase space is not always reducible to four dimensions because no component of the angular momentum needs to be conserved. In principle this requires to analyze motion on five-dimensional iso-energy surfaces.

We shall restrict the discussion to cases where at least one component of the angular momentum is conserved so that the motion can be studied on three-dimensional energy surfaces. A first step to get an overview is to identify bifurcation diagrams in the space of conserved quantities (energy and angular momentum), and to determine the topological nature of the various energy surfaces. This is done by looking at the critical points of an effective potential on the reduced configuration space which we call Poisson torus (rather than Poisson sphere). No further analytical work seems to be possible in general; the rest must be done in terms of Poincaré sections.

It is not trivial to define a “good” 2-D surface of section that (i) captures every trajectory on a given energy surface and (ii) may be represented in projection to the Poisson torus. I will report on the state of affairs and present examples.

## References

- Gashenenko I N and Richter P H 2004 *Int. J. Bif. and Chaos* **14** No. 8, 2525  
Schmidt S, Dullin H R and Richter P H 2008 *arXiv:0803.0076v1*  
<http://arxiv.org/abs/0803.0076>

# New trends in quantum chaos of generic systems

Marko Robnik

*CAMTP - Center for Applied Mathematics and Theoretical Physics,  
University of Maribor, Maribor, Slovenia*

First I shall briefly review the basic elements of the stationary quantum chaos in Hamiltonian systems, the universality classes of energy spectra and eigenfunctions. Then I shall consider the problem of the generic systems whose classical dynamics and the phase portrait is of the mixed type, i.e. regular for certain initial conditions and irregular (chaotic) for other initial conditions. I shall present the so-called Berry-Robnik picture, the Principle of Uniform Semiclassical Condensation (of the Wigner functions of the eigenstates), and the statistical description of the energy spectra in terms of  $E(k,L)$  statistics, which is known and shown to be valid in the semiclassical limit of sufficiently small effective Planck constant and is numerically firmly confirmed. Then I shall show the numerical evidence for the deviations from that prediction in mixed type systems at low energies, due to localization and tunneling effects. The most recently developed random matrix model will be shown to apply in this regime, as it is in very good agreement with numerical and experimental data on the so-called mushroom billiards of Bunimovich. Here are also the important open theoretical questions that I shall address.

## References

- Stöckmann H.-J. 1999 *Quantum Chaos: An Introduction*, Cambridge University Press, Cambridge  
Robnik M 1998 *Nonlinear Phenomena in Complex Systems (Minsk)* **1** 1  
Berry M V and Robnik M 1984 *J. Phys. A: Math. Gen.* **17** 2413  
Gomez J M G, Relano A, Retamosa J, Faleiro E, Salasnich L, Vraničar M and Robnik M 2005 *Phys. Rev. Lett.* **94** 084101  
Robnik M 2006 *International Journal of Bifurcation and Chaos* **16** No.6 1849  
Grossmann S and Robnik M 2007 *Z. Naturforsch.* **62a** 471  
Vidmar G, Stöckmann H.-J., Robnik M, Kuhl U, Höhmann R and Grossmann S 2007 *J.Phys.A: Math.Theor.* **40** 13883  
Bäcker A, Ketzmerick R, Löck S, Robnik M, Vidmar G, Höhmann R, Kuhl U and Stöckmann H.-J. 2008 *Phys.Rev.Lett.* **100** 174103

# Exact analysis of the adiabatic invariants in time-dependent harmonic oscillator

Marko Robnik

*CAMTP - Center for Applied Mathematics and Theoretical Physics,  
University of Maribor, Maribor, Slovenia*

The theory of adiabatic invariants has a long history, and very important implications and applications in many different branches of physics, classically and quantumly, but is rarely founded on rigorous results. It began with the classical paper by Einstein in 1911, following a suggestion by Lorentz in the same year. We treat the general one-dimensional harmonic (linear) oscillator with time-dependent frequency whose energy is generally not conserved, and analyse the evolution of the energy and its statistical properties, like the distribution function of the final energies evolved from an initial microcanonical ensemble. This distribution function turns out to be universal, i.e. independent of the nature of the frequency as a function of time. The theory is interesting from the mathematical point of view as it comprises elements of the theory of dynamical systems, the probability theory and discrete mathematics, and sheds new light on the understanding of the adiabatic invariants in nonautonomous dynamical systems.

## References

- Einstein A 1911 *Inst. Int. Phys. Solway* **1** 450  
Reinhardt W P 1994 *Prog. Theor. Phys. Suppl.* **116** 179  
Henrard J 1993 *Dynamics Reported Vol. 2* Eds. C.K.R.T. Jones, U. Kirchgraber and H.O. Walther, Berlin, Springer, 117  
Landau L D and Lifshitz E M 1996 *Mechanics: Course of Theoretical Physics*, Oxford, Butterworth-Heinemann  
Robnik M and Romanovski V G 2006 *J. Phys. A: Math. Gen* **39** L35-L41  
Robnik M 2005 *Encyclopedia of Nonlinear Science* ed. A. Scott, New York, Routledge, pp2-5  
Robnik M and Romanovski V G 2000 *J. Phys. A: Math. Gen* **33** 5093  
Robnik M and Romanovski V G 2006 *Open Syst. & Infor. Dyn.* **13** 197  
Robnik M, Romanovski V G and Stöckmann H.-J. 2006 *J. Phys. A: Math. Gen* **33** L551  
Kuzmin A V and Robnik M 2007 *Rep. on Math. Phys.* **60** 69-84

# Computational algebra and some applications to differential equations

Valery G. Romanovski

*CAMTP - Center for Applied Mathematics and Theoretical Physics,  
University of Maribor, Maribor, Slovenia*

Consider a system of polynomials

$$f_1(x_1, \dots, x_n) = 0, \dots, f_s(x_1, \dots, x_n) = 0. \quad (7)$$

The set of all solutions to (1) is called the variety. There are numerical algorithms for solving non-linear systems such as (1). These algorithms solve for one solution at a time, and find an approximation to the solution. They ignore the geometric properties of the solutions space (the variety), and do not take into consideration possible alternate descriptions of the variety (using a different system of polynomials). However recently efficient computational algorithms have been developed which enable us to get algebraic and geometric information about the entire solution space of system (1). They are based on the Gröbner bases theory worked out by B.Buchberger around the middle of 60th of last century.

In the lecture we sketch main ideas of the Gröbner bases theory and discuss some algorithms implemented in computer algebra systems (such as Mathematica, Maple and Singular). We consider applications of Gröbner bases to solutions of systems of polynomial equations and to elimination of variables in such systems. We also present a few applications of the algorithms to the study of some problems of the theory of ordinary differential equations.

## References

- Adams W W and Loustaunau P 1994 *An Introduction to Gröbner bases. Graduate Studies in Mathematics.* -V.3 (RI:AMS)
- Buchberger B 1985 in *Multidimensional Systems Theory* ed. Boese N K (D.Reidel Pub. Co.) pp184-232
- Greuel G M, Pfister G and Schönemann H 2005 SINGULAR 3.0. A Computer Algebra System for Polynomial Computations. Centre for Computer Algebra, University of Kaiserslautern. <http://www.singular.uni-kl.de>.
- Cox D, Little J and O'Shea D 1992 *Ideals, Varieties, and Algorithms* (New York: Springer-Verlag)
- Romanovski V G and Shafer D S 2008 *The Center and Cyclicity Problems. A Computational Approach* (Boston: Birkhäuser)

# Self-organized quasiperiodicity in oscillator ensembles with global nonlinear coupling

Michael Rosenblum and Arkady Pikovsky

*Department of Physics,  
University of Potsdam, Potsdam, Germany*

In the classical sense, synchronization of coupled oscillating systems means appearance of certain relations between their phases and frequencies due to weak coupling. After giving brief introduction into the classical theory we review the recent results on self-synchronization in large ensembles of all-to-all interacting units. We discuss experimental examples and the theoretical treatment of the Kuramoto model.

Next, we consider new effects which can appear if the coupling between oscillators is nonlinear in the sense that response of an individual oscillator to a strong driving cannot be taken simply as an “upscaled” response to a weak driving. We formulate a minimal model for an ensemble of nonlinearly coupled oscillators. This model is a generalization of the Kuramoto model. Furthermore, we demonstrate a transition from fully synchronous periodic oscillations to partially synchronous quasiperiodic dynamics. We present an analytic solution of the model that explains the regime where the mean field does not entrain individual oscillators, but has a frequency incommensurate to theirs. The self-organized onset of quasiperiodicity is illustrated with Landau-Stuart oscillators and a Josephson junction array with a nonlinear coupling.

## References

- Kuramoto, Y 1984 *Chemical Oscillations, Waves and Turbulence* (Berlin: Springer)  
Pikovsky A, Rosenblum M and Kurths J 2001 *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge: Cambridge University Press)  
Rosenblum M and Pikovsky A 2007 *Phys. Rev. Lett.* **98** 054102



---

# Some Basic Facts about Logical Circuits

Andreas Ruffing

*Center of Mathematical Sciences, TUM, Munich, Germany*  
*CAMTP - Center for Applied Mathematics and Theoretical Physics,*  
*University of Maribor, Maribor, Slovenia*

This talk is mainly on the pedagogical side of the conference and hence shall also and in particular address all the students attending. Special effect: The intention is to perform also an electric show-experiment, namely Edisons effect, during the talk.

Let us come to the motivation of the topic: In difference equations and discrete dynamical systems theory, the last ten years stood under an impressive sign: Extending results from autonomous difference equations to non-autonomous ones.

Discrete dynamical systems which have only 0 and 1 as a value, may be interpreted as migrating signals of a logical system in which different operations are possible: These operations are the logical circuit operations AND, OR, NOT.

We review main facts of the elementary operations AND, OR, NOT and see how mathematical modelling of a logical circuit works. We will learn about maxterms, minterms and explore an important representation theorem: There is only one kind of logical operator necessary to describe all finite systems of logical circuits: This is the NOR operator. Alternatively a similar representation theorem holds for NAND operators.

All the circuits considered so far are autonomous ones: The same initial signal constellation at the entries of a system leads always to the same results, independent from the position on the time axis.

In the last part of the talk, will we learn about the non-autonomous scenario of RS-logical circuits which allow to store signals and therefore to store information.

In the whole talk, the corresponding physical realization of a logical operator will be presented: Starting from Edisons effect and ending up with modern semiconductor devices.

## References

Kammerer J, Oberthür W, Piegsa J, Siedler H-J *Elektronik III – Grundsaltungen* Edition Pflaum, München 1988

Ruffing A *Logical Circuits as Discrete Dynamical Systems: The Transition from Autonomous Circuits to Nonautonomous Circuits*, preprint in preparation 2008

# Transport and localization of Bose-Einstein condensates in disorder

Peter Schlagheck

*Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany*

In my talk, I will present our research activities on transport processes of interacting Bose-Einstein condensates in dimensionally restricted disorder potentials. We study these processes by numerically integrating the Gross-Pitaevskii equation with an external source that simulates the quasi-stationary injection of coherent matter waves onto the disorder region. For the case of one-dimensional disorder potentials, we find that the interaction between the atoms leads to a cross-over from an exponential to an algebraic decrease of the average transmission with the disorder length, which represents a significant deviation from the scenario of Anderson localization. For two-dimensional disorder potentials, the presence of interaction reverts the phenomenon of weak localization and leads to a cone-shaped dip, instead of a peak, in the angle-resolved current of backscattered atoms. Our numerical findings are corroborated by analytical approaches based, in the 1D case, on transfer-matrix-type methods and, in the 2D case, on nonlinear diagrammatic perturbation theory.

## References

- Paul T, Leboeuf P, Pavloff N, Richter K and Schlagheck P 2005 *Phys. Rev. A* **72** 063621  
Paul T, Schlagheck P, Leboeuf P and Pavloff N 2007 *Phys. Rev. Lett.* **98** 210602  
Hartung M, Wellens T, Müller C A, Richter K and Schlagheck P 2008 *arXiv:0804.3723*

# Solution of Mixed Problem for Elliptic Equation by Monte Carlo and Probability–Difference Methods

Kanat Shakenov

*KazNU - Kazakh National University,  
National Centre of Space Researches and Technologies, Almaty, Kazakhstan*

We consider the following problem in domain  $\Omega \in R_n$  with border  $\partial\Omega$

$$Du(x) \equiv \frac{\partial}{\partial x_i} \left( a_{ij}(x) \cdot \frac{\partial u(x)}{\partial x_j} \right) + b_i(x) \cdot \frac{\partial u(x)}{\partial x_i} + c(x) \cdot u(x) = f(x), \quad \text{in } \Omega, \quad (8)$$

$$\frac{\partial u(x)}{\partial \eta} + d(x) \cdot u(x) = \varphi(x) \quad \text{on } \partial\Omega, \quad (9)$$

where  $\frac{\partial u(x)}{\partial \eta} = a_{ij}(x) \cdot \frac{\partial u(x)}{\partial x_j} \cdot \cos(\vec{n}, x_i)$ . [1].

Coefficients of the operator  $D$  and  $d(x) \geq 0$  are the limited functions,  $f(x) \in H(\Omega)$ . Where  $H(\Omega) \equiv L^2(\Omega)$ . At anyone real  $\xi_0, \xi_1, \dots, \xi_n$  the inequality

$$a_{ij}(x)\xi_i\xi_j - b_i(x)\xi_i\xi_0 - c(x)\xi_0^2 \geq \alpha \sum_{i=1}^n \xi_i^2 + \beta\xi_0^2 \quad (10)$$

is realized,  $\alpha$  and  $\beta$  are positive constants.

The generalized solution  $u(x) \in H^1(\Omega)$  of the problem (8), (9) is an element from  $H^1(\Omega)$  satisfying integral identity

$$D(u, v) + \int_{\partial\Omega} d(x)u(x)v(x)dx = -(f, v) \quad (11)$$

for  $\forall v(x) \in H^1(\Omega)$ . If  $v = u$  in (11), we get

$$D(u, u) + \int_{\partial\Omega} d(x)u^2(x)dx = -(f, u), \quad (12)$$

by virtue of (10) from which follows

$$\int_{\Omega} \left( \alpha u_x^2(x) + \beta u^2(x) \right) dx + \int_{\partial\Omega} d(x)u^2(x)dx \leq -(f, u) \leq \|f\|_{H(\Omega)} \cdot \|u\|_{H(\Omega)}. \quad (13)$$

A priori estimations for the solution  $u(x)$  follows from (13)

$$\|u\|_{H(\Omega)} \leq \frac{1}{\beta} \cdot \|f\|_{H(\Omega)}, \quad \|u_x\|_{H(\Omega)}^2 \leq \frac{1}{\alpha\beta} \cdot \|f\|_{H(\Omega)}^2. \quad (14)$$

Let us consider the case when  $\Omega$  is the bounded domain,  $u \in C^2(\Omega)$  and operator  $D$  given by

$$Du = a_{ij}(x) \frac{\partial^2 u(x)}{\partial x_i \partial x_j} + b_j(x) \frac{\partial u(x)}{\partial x_j} \quad (15)$$

where coefficients  $a_{ij}(x)$ ,  $b_j(x)$  and  $c(x)$  are real measurable functions defined in  $\Omega$ .

Let us assume that  $a_{ij}(x) = a_{ji}(x)$ , that is a matrix of senior coefficients is symmetric.

Let operator  $D$  acting in  $C^2(\Omega)$  by (15) is elliptic, that is for  $\exists \alpha > 0$ ,  $\forall (\xi_1, \dots, \xi_n) \in R^n$  the inequality  $a_{ij}(x)\xi_i\xi_j \geq \alpha|\xi|^2 \equiv \xi_i^2$  is correct.

Let  $f$  is measurable functions in  $\Omega$ . The function  $u \in C^2(\Omega)$  is called as the regular solution of the equation  $Du(x) = f(x)$ , if it satisfies to this equation in each point  $x \in \Omega$ . Closed domain  $\bar{\Omega}$  belongs to the  $A^{(k,\lambda)}$  class,

if in vicinity of each point  $x \in \partial\Omega$   $\partial\Omega$  is given by  $\xi_n = \gamma(\xi_1, \dots, \xi_{n-1})$  in some system of coordinates,  $\gamma$  has  $k$  continuous derivatives, and also  $\gamma^{(k)} \in C^{(0,\lambda)}$ , that is it satisfies to Holder condition with index  $\lambda$ . [2].

For continuous function  $\varphi(x)$  on the border  $\partial\Omega$ , measurable  $f(x)$  and elliptic operator  $D$  we consider the mixed problem

$$Du(x) = f(x), \quad x \in \Omega, \quad (16)$$

$$\frac{\partial u(x)}{\partial \eta} + d(x)u(x) = \varphi(x), \quad x \in \partial\Omega. \quad (17)$$

Let us assume that  $a_{ij} \in C^{(1)}(\Omega)$ .

Let  $e_i = b_i - \sum_{j=1}^n \frac{\partial a_{ij}}{\partial x_j}$ , then

$$Du = \frac{\partial}{\partial x_j} \left( a_{ij} \frac{\partial u}{\partial x_i} \right) + e_i \frac{\partial u}{\partial x_j} + cu.$$

If  $e_i \in C^{(1)}(\Omega)$ , then operator  $D^*$  acting at function  $v(x) \in C^{(2)}(\Omega)$  by equation

$$D^*u = \frac{\partial}{\partial x_j} \left( a_{ij} \frac{\partial v}{\partial x_i} \right) + e_i \frac{\partial v}{\partial x_i} + cv$$

called as conjugate to  $D$ .

Let  $\mathcal{L}(y, x)$  is Levi function,  $T(x) \subset \Omega$ ,  $T(x)$  is a family of domains, for example a family of balls of maximal radius with center in  $x$ . Let us assume  $\mathcal{L}(y, x) = 0$  for  $y \in \partial T$ , then

$$u(x) = \int_T \left( u D_y^* \mathcal{L} + \mathcal{L} f \right) dy - \int_{\partial T} a_{ij}(y) \frac{\partial \mathcal{L}}{\partial y_i} n_i u(y) d_y S. \quad (18)$$

The integral equation (18) can be solved by Monte Carlo methods, if the integral operator  $K$  of this equation satisfies to condition

$$\|K\|_{H^\infty(\Omega)} = \sup_{x \in \Omega} \int_{T(x)} k(x, y) dy < 1, \quad (19)$$

where  $k(x, y)$  is a node of the integral equation, it depends on  $a_{ij}(x)$ , on a derivative  $\frac{\partial \mathcal{L}}{\partial y_i}$  and on component of an external normal to border of the domain  $T$ . [2].

At performance of the condition (19) the integral equation (18) can be solved by "random walk by spheres" and "random walk by balls" algorithm of Monte Carlo methods, also it is possible to construct the  $\varepsilon$ -displaced estimations for  $u(x)$ . For example, such case is possible, if coefficients of the operator  $D$  are constant.

Let  $\partial\Omega$  is a surface of Lyapunov, the surface  $\Omega$  is convex, coefficients of the operator  $D$  are constant. Then the norm of the integral operator acting in  $C(\bar{\Omega})$  less than 1. Hence, it is possible to apply Neumann-Ulam scheme to the equation (18).

The integral equation (18) is solved by "random walk by balls" algorithm of Monte Carlo methods. By reaching  $\varepsilon$ -boundary Markov chain reflected with the probability  $p_{\partial\Omega_\varepsilon} = \frac{|a_{ij}|}{|a_{ij}|+|d|}$  and adsorbed with the probability  $q_{\partial\Omega_\varepsilon} = \frac{|d|}{|a_{ij}|+|d|}$ .

At transition from one condition to the following condition the "weight" of node, that is defined by the recurrence relation

$$Q_0 = 1, \quad Q_{i+1} = Q_i \frac{k(x_i, x_{i+1})}{p_{\Omega}(x_i, x_{i+1})}, \quad i = 0, 1, \dots,$$

is taken into account. On a border the "weight" of border proportional

$$Q_{\partial\Omega} = \frac{\varphi(x_i)}{|a_{ij}| + |d|}$$

is taken into account.

Let us denote by  $h$  a step of the difference scheme in each coordinate direction and by  $e_i$  coordinate unit vector in  $i$ -th coordinate direction. We approximate the domain  $\Omega$  and operator  $D$  by finite difference method.

Let us define

$$G_h(x) = 2 \sum_i a_{ii}(x) - \sum_{i,j, i \neq j} |a_{ij}(x)| + h \sum_i |b_i(x)|$$

and assume that

$$a_{ii}(x) - \sum_{j, j \neq i} |a_{ij}(x)| > 0, \quad i = 1, 2, \dots, n.$$

Then we define

$$\Delta t^h(x) = \frac{h^2}{G_h(x)},$$

$$p^h(x, x \pm e_i h) = \frac{a_{ii}(x) - \sum_{j, j \neq i} |a_{ij}(x)| + h^2 b_i^\pm(x)}{G_h(x)},$$

$$p^h(x, x + e_i h \pm e_j h) = \frac{a_{ij}^\pm(x)}{G_h(x)},$$

$$p^h(x, x - e_i h \pm e_j h) = \frac{a_{ij}^\pm(x)}{G_h(x)},$$

$p^h(x, y) = 0$  for the others  $x, y \in \Omega \in R_h^n$ . Function  $p^h(x, y)$  is nonnegative, the sum on  $y$  is equal 1 for each  $x$ . This means that  $p^h(x, y)$  is the probability of transitions of some Markov chain that we denote by  $\{\xi_n^h\}$ .  $p^h(x, y)$  will be coefficients in finite difference approximation. **It is easy to prove that to finite difference mixed problem the Neumann–Ulam scheme is applicable.**

We'll divide a discrete border into the reflecting  $\partial\Omega_R^h$  and the absorbing  $\partial\Omega_A^h$ , then it is possible to construct the  $\varepsilon$ -displaced approximation of the unique decision in the point  $x$ . For example, it is possible to construct Markov chain by "random walk by lattices" and to define  $\{\xi_n^h\}$  along this chain. [3], [4], [5], [6], [7], [8].

## References

1. Ladyzhenskaja O.A., *Boundary problem of mathematical physics*. Nauka, Moscow, 1973. (in Russian)
2. Miranda C. *Partial Differential Equations of Elliptic Type*. Springer – Verlag, New York, 1970, pp. 370
3. Kushner Harold J., *Probability Methods of Approximations in Stochastic Control and for Elliptic Equations*. Academic Press, New York – San-Francisco – London, 1977.
4. Haji–Sheikh A. and Sparrow E.M., The floating random walk and its application to Monte Carlo solutions of heat equations. *SIAM Journal on Applied Mathematics*, Vol. 14, No. 2, March, 1966. Printed in U.S.A. P. 370–389.
5. Haji–Sheikh A., Application of Monte Carlo methods to thermal conduction problems. Ph.D. Thesis, University of Minnesota, Minneapolis, 1965.
6. Shakenov K., Solution of one mixed problem for equation of a relaxational filtration by Monte Carlo methods. *Notes on Numerical Fluid Mechanics and Multidisciplinary Design*, Springer Berlin /Heidelberg. Volume 93/2006. Book *Advances in High Performance Computing and Computational Sciences*. Chapter 71. Pages 205 – 210.
7. Shakenov K.K., Solution of problem for one model of relaxational filtration by probability-difference and Monte Carlo methods. Polish Academy of Sciences. Committee of Mining. *Archives of Mining Sciences*, Volume 52, Issue 2, Krakow, 2007. Pages 247 – 255.
8. Shakenov K., Solution of one problem of linear relaxational filtration by Monte Carlo methods. *International Conference on COMPUTATIONAL MATHEMATICS (ICCM 2002)*, Part I. ICM&MG Publisher. Novosibirsk, Russia, 2002. P. 276 – 280.

# Amphibious complex orbits and its manifestation in quantum mechanics

Akira Shudo

*Department of Physics, Tokyo Metropolitan University, Tokyo Japan*

The wavepacket in quantum mechanics has a nonzero transition amplitude to arbitrary regions even when the initial and final regions are separated by energetic or dynamical barriers. Such an aspect has something in common to *classically ergodic systems* in which orbits are allowed to come close to any point in phase space. Apparently, this would be merely an analogy and one may think that these are entirely unrelated to each other since the former is a result of an intrinsic dynamical property but the latter is due to purely quantum effects like tunneling.

However, the studies on complex dynamical systems in more than one dimension have revealed there exists a unique ergodic measure in complex phase space, which has mixing property and thus ergodic (Bedford & Smillie 1991a, 1991b, 1992). A remarkable fact is that it holds not only in hyperbolic systems but also in mixed systems in which quasiperiodic or chaotic orbits coexist in phase space.

These mathematical results predict that the transition between classically disconnected regions occurs under the same mechanism as in classically ergodic systems, and that the orbits connecting classically disjointed components should have *amphibious* nature (Shudo *et al* 2008a, 2008b).

Here, we show that amphibious complex trajectories explain the existence of *amphibious states* which were found as the quantum states ignoring underlying classical invariant structures (Hufnagel *et al* 2002, Bäcker *et al* 2005). We also present the results of numerical investigations for a coupled kicked oscillator, in which the amphibious states are realized without a special tuning of the system (Ishikawa *et al* 2007, 2008).

## References

- Bäcker A, Ketzmerick R and Monastera A.G 2005 *Phys.Rev.Lett.***94** 054102.  
Bedford E and Smillie J 1991a *Invent. Math.* **103** 69-99.  
Bedford E and Smillie J 1991b *J. Amer. Math. Soc.* **4** 657-679.  
Bedford E and Smillie J 1992 *Math. Ann.* **294** 395-420.  
Hufnagel L, Ketzmerick R, Otto MF and Schanz H 2002 *Phys.Rev.Lett.***87** 154101.  
Ishikawa A, Tanaka A and Shudo A 2007 *J. Phys. A* **40** F1-F9.  
Ishikawa A, Tanaka A and Shudo A 2008 in preparation.  
Shudo A, Ishii Y and Ikeda K S 2008a *EuroPhys.Lett.* **81** 50003.

# Toward pruning front theory for the Stokes geometry

Akira Shudo

*Department of Physics, Tokyo Metropolitan University, Tokyo Japan*

The saddle point method is known to be an efficient technique to approximately evaluate integrals. A difficulty in applying the saddle point method in general is that not all of the saddles point solutions necessarily contribute. The *Stokes geometry* carries all geometrical information to judge which saddles should appear in its evaluations and which should be dropped, thereby it tells us how to construct global asymptotic solutions from the local pieces.

Here we present an idea for developing bifurcation theory of the Stokes geometry in quantum maps whose classical counterpart exhibit chaos (Shudo 2007, Shudo and Ikeda 2008). A concrete recipe to give the complete Stokes geometry is presented for the quantum Hénon map, in which some new ingredients absent in conventional theory of asymptotics (Aoki et al 1994, 2005a, 2005b, Howls 2004) are taken into account.

A key strategy is to first establish the Stokes geometry in the horseshoe limit, in which the underlying classical dynamics is described by the binary symbolic dynamics and the corresponding Stokes geometry is also analytically tractable, and then to trace it as a function of the system parameter by focusing on bifurcation of the Stokes geometry. The approach exactly follows the *pruning front theory* of classical symbolic dynamics (Cvitanović et al 1988).

## References

- Aoki T, Kawai T and Takei Y 1994 *Analyse algébrique des perturbations singulières. I.* (ed. by L. Boutet de Monvel, Hermann) 69-84.  
Aoki T, Kawai T, Sasaki S, Shudo A and Takei Y 2005a *J.Phys.* **A38** 3317-3336.  
Aoki T, Kawai T, Koike T, Sasaki S, Shudo A and Takei Y, 2005b *RIMS Kokyuroku* **1424** 53-63.  
Cvitanović P, Gunaratne GH and Procaccia I 1988 *Phys.Rev* **A38** 1503-1520.  
Howls CJ, Langman PJ and Olde Daalhuis AB 2004 *Proc.R.Soc.London* **A460** 2285-2303.  
Shudo A 2007 *Algebraic analysis of differential equations —from microlocal analysis to exponential asymptotics — Festschrift in Honor of Takahiro Kawai*, eds. T. Aoki, H. Majima, Y.Takei, N. Tose (Springer) 251-264.  
Shudo A and K.S. Ikeda 2008 *Nonlinearity* in press.

# Coupled oscillators: Why may they be used to describe cardiovascular dynamics?

Aneta Stefanovska

*Department of Physics, Lancaster University, Lancaster, UK  
Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia*

For a healthy human in repose the heart expels blood about once per second in an oscillatory manner. An amount of blood equivalent to the total amount (5.0–5.5 l) is circulated in about one minute. During this interval, which may be considered as an average circulation time for blood around the human body, five oscillatory components have been observed. They correspond to control mechanisms that reduce the velocity of flow, enabling the cells that surround the capillary bed to exchange energy and matter efficiently with the blood, thereby realizing the ultimate goal of the circulation. At the moment, this story, can be reconstructed only partially, because –

- (i) The notion of flow is meaningful only at the macroscopic level, where many processes coexist and interact, whereas microscopic studies of the mechanisms of vasomotion are of limited use.
- (ii) Non-invasive techniques to record blood circulation in small vessels (based on laser Doppler measurements) facilitate recordings of the skin flow. At greater depths, only the flow in the larger vessels can be assessed (by ultrasound techniques).
- (iii) Currently available methods of time series analysis allowing for reconstruction of time-variable complex dynamics are limited in efficacy.

The development of data analysis methods for complex oscillatory dynamics following Grassberger and Proccacia's (1983) algorithm for calculation of the correlation dimension will be reviewed, and the advantages and pitfalls of the currently used methods will be discussed. In addition, the possibilities of the phase dynamics approach for reconstructing the dynamics properties of living systems will be considered, as will also the importance of the uncertainty principle. Methods for the detection of synchronization and direction of coupling, and the importance of surrogate data in hypothesis testing, will be outlined. The current state-of-the art understanding of the cardiovascular system as a system of coupled oscillators will be reviewed.

## References

- Grassberger P, Proccacia I 1983 *Phys Rev Lett* **50** 346–349.  
Haken H 1975 *Rev Mod Phys* **47** 67–121.  
Kuramoto Y 1984 *Chemical Oscillations, Waves, and Turbulence*, Springer, New York.  
Pikovsky A, Rosenblum M, Kurths J 2001 *Synchronization – A Universal Concept in Nonlinear Sciences*, CUP, Cambridge.  
Paluš M 2008 *Contemp Phys*, in press.  
Stefanovska A, Bračič M 1999 *Contemp Phys* **40** 31–55.  
Stefanovska 2007 *IEEE Eng Med Biol Magazine* **26** 25–29.



---

# Can the brain and the cardiovascular system talk to each other and, if so, how?

Aneta Stefanovska

*Department of Physics, Lancaster University, Lancaster, UK  
Faculty of Electrical Engineering, University of Ljubljana, Ljubljana, Slovenia*

The role of the central nervous system in controlling the cardiovascular system has been discussed in numerous studies. The basic functional unit of the nervous system – the action potential – has been understood in great detail. In their pivotal experiments, however, Hodgkin, Huxley and Katz (1952) clamped the membrane potential at a constant value so that, ever since, its temporal dynamics was neither investigated nor properly understood. *In vivo*, the ionic concentrations fluctuate continuously as part of the normal function of the cardiovascular system. Recently developed non-invasive imaging techniques allow for understanding of the role of glial cells in mediating the ionic concentrations of the neurones in the brain (Schipke *et al*, 2008), thereby opening up the question of dynamical interactions between the cardiovascular oscillations and the brain waves.

We will review recent studies based on the phase dynamics approach that have began to illuminate possible bidirectional causal interactions between the two systems. We show that the  $\delta$  waves extracted from an EEG signal, and the phase of respiration, are coupled differently during deep and light anaesthesia (Musizza *et al*, 2007). Moreover, using fMRI, a characteristic frequency of 0.03-0.04 Hz was demonstrated in the brain (Horovitz *et al*, 2008). This frequency was already known to exist in the cardiovascular system. The characteristic frequencies of these two major systems of the human organisms – the cardiovascular that takes care of energy and matter exchange, and the neuronal system that takes care of information transfer within the body – will be summarized, pointing to the new opportunities opened up by the phase dynamics approach for reaching an understanding of how the two system interact so efficiently.

## References

- Hodgkin AL, Huxley AF 1952 *J Physiol-London* **117** 500–544.  
Hodgkin AL, Huxley AF, Katz B 1952 *J Physiol-London* **116** 424–448.  
Musizza B, Stefanovska A, McClintock PVE, Paluš M, Petrovčič J, Ribarič S, Bajrović FF *J Physiol-London* **580** 315–326.  
Schipke CG, Heidemannb A, Skupin A, Peters O, Falcke M, Kettenmann H 2008 *Cell Calcium* **43** 285-295.  
Horovitz SG, Fukunaga M, de Zwart JA, van Gelderen P, Fulton SC, Balkin TJ, Duyn JH 2008 *Human Brain Mapping* **29** 671-682.

# Microwave studies of chaotic systems

## Lecture 1: Currents and vortices

**Hans-Jürgen Stöckmann**

*Fachbereich Physik,  
Philipps-Universität Marburg, Marburg, Germany*

In flat quasi-two-dimensional microwave resonators there is a one-to-one correspondence between the Helmholtz equation for the electric field pointing from the bottom to the top, and the stationary Schrödinger equation for the corresponding quantum-mechanical billiard system. This offers the unique possibility to test theories, originally developed for quantum-mechanical systems, in their microwave analogue (Stöckmann 1999). I shall illustrate these features in a series of three lectures.

In closed systems the wave functions are characterized by meandering patterns of nodal lines and domains, the universal features of which can be well described by percolation theory. In open systems there are no longer nodal lines, but only nodal points, corresponding to vortices of the quantum-mechanical flow. Here the random plane wave approach has proven to be a powerful method to describe the universal features of the wave fields. The model allows for an easy calculation of various distribution functions of currents, vortex densities etc., as well as various pair correlation functions. In microwave experiments a large number of predictions from theory could be verified (Kuhl et al. 2007).

In systems with a potential landscape the model is no longer applicable. Here the formation of caustics is possible, leading to branch-like structures in the flow as observed on scanning tunneling microscopy (STM) experiments on quantum point contact structures (Topinka et al. 2001). In microwave billiards potentials may be realized by introducing appropriately shaped scatterers into the resonator. By this the STM results could be qualitatively confirmed. The very same model has been also used for the formation of wave patterns in the sea due to locally varying velocity fields (Heller et al. 2008), thus allowing at the same time for the study of non-Gaussian wave patterns (rogue waves etc.) in the ocean.

### References

- Stöckmann H-J 1999 *Quantum Chaos - An Introduction* (University Press)  
Kuhl U 2007 *Eur. Phys. J. Special Topics* **145** 103  
Topinka M A *et al.* 2001 *Nature* **410**, 183  
Heller E J, Kaplan L and Dahlen A 2008 *nlin.CD/08010613*

# Microwave studies of chaotic systems

## Lecture 2: Line width distributions in open systems

**Hans-Jürgen Stöckmann**

*Fachbereich Physik,  
Philipps-Universität Marburg, Marburg, Germany*

In a typical microwave experiment from the transmission between or reflection at various antennas the system properties are to be determined. There is the obvious question how to distinguish between the system properties and those of the connecting cables, antenna connectors etc.. Scattering theory, originally developed in nuclear physics, is the method of choice to cope with this situation (Guhr et al. 1998). The scattering matrix is given by

$$S(E) = 1 - 2iW^\dagger \frac{1}{E - H_{\text{eff}}} W \quad (20)$$

where  $H_{\text{eff}} = H - iWW^\dagger$ . The diagonal elements  $S_{ii}$  of the scattering matrix are the reflection amplitudes at the  $i$ th antenna, whereas the non-diagonal elements  $S_{ij}$  are the transmission amplitudes between antenna  $i$  and  $j$ .  $H$  is the Hamiltonian of the unperturbed system, and the  $W$  matrix contains the information of the coupling of the  $i$ th antenna to the  $n$ th wave function. There is a vast amount of theoretical studies (Beenakker 1998), as well as a number of microwave results (Kuhl et al. 2005). Up to now, however, apart from one exception (Persson et al. 2000), only average quantities such as the distribution of reflection or transmission coefficients have been studied. The poles of the scattering matrix had not been accessible, however, in the regime of strong overlap. Here recently a breakthrough has been achieved. By an application of the method of harmonic inversion (Wiersig, Main 2008) the poles of the scattering matrix could be resolved in a regime, where the line widths exceed the mean level spacings by a factor of 10 and even more. By this it became possible to test theories on the distribution of line widths in chaotic systems (Sommers et al. 1998), which seemed to be unaccessible experimentally up to now.

### References

- Guhr T, Müller-Groeling A and Weidenmüller H. A. 1998 *Phys. Rep.* **299**, 189  
 Beenakker C. W. J. 1998 *Phys. Rev. Lett.* **81**, 1829  
 Kuhl U, Stöckmann H-J and Weaver R 2005 *J. Phys. A* **38** 10433  
 Persson E, Rotter I, Stöckmann H-J and Barth M 2000 *Phys. Rev. Lett.* **85**, 2478  
 Wiersig J and Main J 2008 *Phys. Rev. E* **77** 036205  
 Sommers H J, Fyodorov Y V and Titov M *J. Phys. A* **32**, L77

# Microwave studies of chaotic systems

## Lecture 3: A random matrix approach to fidelity

Hans-Jürgen Stöckmann

*Fachbereich Physik,  
Philipps-Universität Marburg, Marburg, Germany*

The concept of fidelity has been introduced more than 20 years ago by Peres (1984) as a quantitative measure to characterize the quantum-mechanical stability of a system against perturbations. The fidelity amplitude is defined as the overlap integral of a wave packet  $|\psi_0\rangle$  with itself, after the evolution under two slightly different Hamiltonians,

$$f(t) = \left\langle \psi_0 \left| e^{\frac{i}{\hbar}(H+\lambda V)t} e^{-\frac{i}{\hbar}Ht} \right| \psi_0 \right\rangle \quad (21)$$

where  $\lambda V$  is a small perturbation of  $H$ . There had been a renewed interest in fidelity because of its obvious relevance for quantum computing.

Most perturbations studied up to now had been global, which can be treated theoretically by taking  $H$  and  $V$  from one of the Gaussian ensembles. In billiards a global perturbation can be achieved, e. g., by the shift of one wall. Local perturbation had been overlooked, which is surprising, since most, if not all, perturbations in real systems are of the local type, such as a diffusive jump of an atom, or a spin-flip. Only recently the fidelity decay due to a local perturbation has been studied as well, realized in a microwave billiard by the shift of a small scatterer (Höhmman et al. 2008). For local perturbations it is easy to calculate the fidelity decay analytically, which shows up to be algebraically, again by using the random plane wave approach.

For global perturbations supersymmetry techniques can be used to calculate the Gaussian averages analytically (Stöckmann, Schäfer 2005). In this case the long-time decay is either Gaussian or exponential. On this occasion a short tutorial introduction into the concept of supersymmetry will be given. We shall see that the underlying ideas are simple, and often exact results are easy to obtained. Unfortunately mostly the resulting integrals are very complicated, and much additional effort is needed to bring the solution, though exact, into a form which is suitable for practical purposes.

### References

- Peres A 1984 *Phys. Rev. A* **30**, 1610  
Höhmman R, Kuhl U and Stöckmann H-J 2008 *Phys. Rev. Lett.* **100**, 124101  
Stöckmann H-J and Schäfer R 2005 *Phys. Rev. Lett.* **94**, 244101

---

# On the mechanism of quantization of chaos.

## - Phase quantization -

Kazuo Takatsuka

*Department of Basic Science, Graduate School of Arts and Sciences,*

*The University of Tokyo, Komaba, Tokyo, Japan*

Since the early stage of the study of Hamilton chaos, semiclassical quantization based on the low-order Wentzel-Kramers-Brillouin (WKB) theory, the primitive semiclassical approximation to the Feynman path integrals (or the so-called Van Vleck propagator), and their variants have been suffering from difficulties such as divergence in the correlation function, and so on. Even the celebrated Gutzwiller trace formula is not an exception. It is widely recognized that the essential drawback of these semiclassical theories commonly originates from the erroneous feature of the amplitude factors in their applications to classically chaotic systems. This makes a clear contrast to the success of the Einstein-Brillouin-Keller quantization condition for regular (integrable) systems. We show here that energy quantization of chaos in semiclassical regime (characterized with a small Planck constant) is, in principle, possible in terms of constructive and destructive interference of phases alone, and the role of the semiclassical amplitude factor is indeed negligibly small, as long as it is not highly oscillatory [1]. To do so, we first sketch the mechanism of semiclassical quantization of energy spectrum with the Fourier analysis of phase interference in a time correlation function, from which the amplitude factor is practically factored out due to its slowly varying nature [2]. In this argument there is no distinction between integrability and nonintegrability of classical dynamics. Then we present numerical evidences that chaos can be indeed quantized by means of amplitude-free quasi-correlation functions. This is called phase quantization [1]. Finally, we show explicitly that the semiclassical spectrum is quite insensitive to smooth modification (rescaling) of the amplitude factor [3]. At the same time, we note that the phase quantization naturally breaks down when the oscillatory nature of the amplitude factor is comparable to that of the phases, which is quite likely to be materialized in a very high energy case and/or in dynamics on a hard potential function like the stadium billiard. Such a case generally appears when the Planck constant of a large magnitude pushes the dynamics out of the semiclassical regime. We will show a possible route to such a regime beyond the standard WKB theory.

- [1] Takahashi S and Takatsuka K 2007 *J. Chem. Phys.* **127**, 084112.
- [2] Ushiyama H and Takatsuka, K 2005 *J. Chem. Phys.* **122**, 224112.
- [3] Yamashita T and Takatsuka K 2007 *Prog. Theoret. Phys. Suppl.* **161**, 56.

# Quantum wavepacket bifurcation and entanglement in molecules. - Generalization of classical mechanics -

Kazuo Takatsuka

*Department of Basic Science, Graduate School of Arts and Sciences,  
The University of Tokyo, Komaba, Tokyo, Japan*

I discuss characteristic dynamics of a system consisting of classical and quantum subsystems strongly coupling with each other. Typically, the quantum subsystem is composed of light and fast particles, while heavy and slow particles constitute the classical subsystem. Such systems are quite ubiquitous, since molecules are more or less approximated in terms of this mixed quantum-classical representation, with nuclei being heavy and classical particles and electrons being light and quantum particles. More explicitly, the separation of electronic and nuclear motions based on this idea constitutes the heart of the so-called Born-Oppenheimer approximation, which is now the standard concept in material sciences. In the Born-Oppenheimer view, nuclei are supposed to propagate in time (either classically or quantum mechanically) on a potential energy function, which is given as electronic energy at parametrically fixed nuclear coordinates.

We first examine the validity range of the Born-Oppenheimer (BO) approximation with respect to the variation of the mass ( $m$ ) of negatively charged particle by substituting an electron ( $e$ ) with muon ( $\mu$ ) and antiproton ( $\bar{p}$ ) in hydrogen molecule cation [1]. With use of semiclassical quantization applied to  $(ppe)$ ,  $(pp\mu)$ , and  $(pp\bar{p})$  under a constrained geometry, we estimate the energy deviation of the non-BO vibronic ground state from the BO counterpart. It is found that the error in the BO approximation scales to the power of 3/2 to the mass of negative particles, that is,  $m^{1.5}$ . The origin of this clear-cut relation is analyzed based on the original perturbation theory due to Born and Oppenheimer, and it is shown the first correction to the BO approximation should arise from the sixth order term that is proportional to  $m^{6/4}$ .

We next proceed to the breakdown of the Born-Oppenheimer approximation, in which two (or more) adiabatic potential energy functions undergo near degeneracy (the so-called avoided crossing). In the vicinity of this avoided crossing, the electronic and nuclear motions strongly couple and thereby entangle. In passing this region, both the electronic and nuclear wavepackets bifurcate in the standard manner of quantum entanglement. We show such wavepacket bifurcation can be directly observed in terms of the femtosecond pump-probe photoelectron spectroscopy. As an illustrative example, we show a (theoretically mapped) movie of such a wavepacket bifurcation arising from the nonadiabatic (non-Born-Oppenheimer) motion on the ionic ( $\text{Na}^+\text{I}^-$ ) and covalent ( $\text{NaI}$ ) states of sodium iodide [2].

Classical trajectory study of nuclear motion on the Born-Oppenheimer potential energy surfaces is now one of the standard methods of molecular dynamics (as in the simulation of protein dynamics). However, as soon as more than a single potential energy surface are involved due to nonadiabatic coupling as above, such a naive application of classical mechanics loses its theoretical foundation. This is a classic and fundamental issue in the foundation of molecular and material science. To cope with this problem, we propose a generalization of classical mechanics, in which the force appears as a matrix with the dimension equal to the number of the adiabatic states involved. With this matrix force, the "classical" paths passing across the avoided crossing region undergo branching associated with the simultaneous electronic wavepacket bifurcation, thereby making the quantum-classical description of entanglement possible [3]. Numerical examples along with relevant graphics will be presented in a realistic molecular system.

## References

- [1] Takahashi S and Takatsuka K 2006 *J. Chem. Phys.* **124** 144101.
- [2] Arasaki Y, Takatsuka K, Wang Kwanghsi, and McKoy K 2003 *Phys. Rev. Lett.* **90** 248303.
- [3] Takatsuka K 2007 *J. Phys. Chem. A* **111** 10196.

# Model based development of desynchronizing brain stimulation techniques I: Reshaping neural networks

Peter A. Tass

*Institute for Neuroscience and Biophysics 3 - Medicine,  
Research Center Jülich, Jülich, Germany*

Within the last years standard high-frequency (HF) deep brain stimulation became the standard therapy for medically refractory movement disorders. HF deep brain stimulation has been developed empirically, mainly based on observations during neurosurgical procedures. In contrast, to overcome limitations of standard HF deep brain stimulation, we use a model based approach. We make mathematical models of affected neuronal target populations and use methods from statistical physics, nonlinear dynamics, and synergetics to develop mild and efficient control techniques (Tass 1999; 2003; Hauptmann et al. 2007a, 2007b; Tass et al. 2008; Popovych et al. 2005; Omel'chenko et al. 2008). We specifically utilize dynamical self-organization principles and plasticity rules. In this way, we have developed multi-site coordinated reset (MCR) stimulation, an effectively desynchronizing brain stimulation technique (Tass 2003). The goal is not only to counteract pathological synchronization on a fast time scale, but also to unlearn pathological synchrony by therapeutically reshaping neural networks (Tass & Majtanik 2003). We examined the effects of MCR stimulation in 20 patients with severe Parkinson's disease or essential tremor during the first week after electrode implantation with a novel portable brain stimulator. According to our theoretical predictions, in all 20 patients epochs of MCR stimulation caused pronounced therapeutic effects, which outlasted MCR stimulation during the whole post-MCR observation period prior to dismissal (i.e. during at least four days).

## References

- Tass PA Phase resetting in medicine and biology: Stochastic modelling and data analysis Springer Verlag, Berlin (1999)
- Tass PA 2003 *Biol. Cybern.* **89** 81-88
- Tass PA, Majtanik M 2006 *Biol. Cybern.* **94** 58-66
- Hauptmann C, Popovych OV, Tass PA 2007a, *Expert Rev Med Devices* **4** 633-650
- Tass PA, Hauptmann C and Popovych OV 2008: Brain pacemaker. *Encyclopedia of Complexity and System Science*. (Springer Berlin Heidelberg New York.) In press
- Popovych OV, Hauptmann C and Tass PA 2005 *Phys. Rev. Lett.* **94** 164102
- Hauptmann C et al. 2007b *Phys. Rev. E* **76** 066209
- Omel'chenko O, Maistrenko Y and Tass PA 2008 *Phys. Rev. Lett.* **100** 044105

# Model based development of desynchronizing brain stimulation techniques II: Target point diagnosis and restoring physiological connectivity

Peter A. Tass

*Institute for Neuroscience and Biophysics 3 - Medicine,  
Research Center Jülich, Jülich, Germany*

High-frequency test stimulation for tremor suppression is a standard procedure for functional target localization during deep brain stimulation. This method does not work in cases where tremor vanishes intraoperatively, e.g. due to global anesthesia or due to an insertional effect. To overcome this difficulty, we developed a stimulation technique that effectively evokes tremor in a well-defined and quantifiable manner. For this, we used patterned low-frequency stimulation (PLFS), i.e. brief high-frequency pulse trains administered at pulse rates similar to the neurons' preferred burst frequency (Barnikol et al. 2008). In a computational investigation of an oscillatory neuronal network temporarily rendered inactive, we found that already at weak stimulus intensities PLFS evokes synchronized activity, phase locked to the stimulus. We applied PLFS to a patient with spinocerebellar ataxia type 2 (SCA 2) with pronounced tremor that disappeared intraoperatively under general anesthesia. In accordance with our computational results, PLFS evoked tremor, phase locked to the stimulus (Barnikol et al. 2008). In particular, weak PLFS caused low-amplitude, but strongly phase-locked tremor (Barnikol et al. 2008). PLFS test stimulations provided the only functional information about target localization. Optimal target point selection was confirmed by excellent postoperative tremor suppression (Freund et al. 2007; Barnikol et al. 2008). The clinical results obtained in our SCA 2 patient with permanent high-frequency stimulation are in accordance with a computational study (Hauptmann & Tass 2007). Additionally taking into account physiological input into neuronal populations, we revealed that multi-site coordinated reset stimulation is able to restore physiological connectivity (in preparation). Implications thereof will be explained in detail.

## References

- Barnikol UB, Popovych OV, Hauptmann C, Sturm V, Freund H-J, Tass PA 2008 *Phil. Trans. Roy. Soc. B* In press
- Freund H-J, Barnikol UB, Nolte D, Treuer H, Auburger G, Tass PA, Samii M, Sturm V 2007 *Mov. Dis.* **22** 732-735
- Hauptmann C, Tass PA 2007 *Biosystems* **89** 173-181



---

# Dynamical Reaction Theory

Mikito Toda

*Nara Women's University, Nara, 630-8506, Japan*

We will discuss the dynamical reaction theory for multi-dimensional Hamiltonian systems. Roughly speaking, the dynamical processes of reactions consist of the following three, i.e., redistribution of energy among vibrational modes in the potential well, going over the potential saddles, and their dynamical connection. For each of the former two processes, perturbation approach is possible to construct the normal form. For the distribution, the normal form describes how the vibrational modes exchange energy in the well. When nonlinear resonance takes place, the perturbation theory breaks down, and energy exchange among vibrational modes is enhanced dramatically there. In the action space, resonant regions constitute the network called the Arnold web. Therefore, properties of the Arnold web play an important role in our topics. For the processes of going over the saddle, the normal form theory is developed recently, which provides a mathematically sound foundation of the concept of transition states (TSs). The theory is based on the phase space structures called normally hyperbolic invariant manifolds (NHIMs). It enables us to define the boundary between the reactant and the product, and to single out the reaction coordinate near the saddles of index one.

After reviewing these concepts, we discuss the following three subjects. First, we present existence of fractional behavior for processes in nonuniform Arnold webs. We also discuss reaction processes under laser fields utilizing cooperative effects of laser fields and the Arnold web. This offers a possibility of manipulating reaction processes by designing laser fields. Second, We discuss that the conventional idea of reaction coordinates is not valid when the condition of normal hyperbolicity is broken. These situations take place when the values of tangential Lyapunov exponents on the NHIMs become comparable to those of the normal exponents. Third, we discuss the network of intersections between stable/unstable manifolds emanating from the NHIMs in multi-dimensional Hamiltonian systems. We point out that the network enables the chaotic itinerancy in Hamiltonian dynamical systems.

## References

- M. Toda, Adv. Chem. Phys. **123**, 153 (2002).  
M. Toda, Adv. Chem. Phys. **130A**, 337 (2005)  
C-B. Li, Shojiguchi, M. Toda and T. Komatsuzaki, Phys. Rev. Lett., **97**, 028302(2006).  
A. Shojiguchi and C. B. Li and T. Komatsuzaki and M. Toda, Phys. Rev., **E78**, 056205(2007).

# Granular Matter Dynamics

## I. The Many Phases of Vibrated Granular Matter

Ko van der Weele

*Mathematics Department,  
University of Patras,  
26500 Patras, Greece*

Vibrated granular matter can behave in an amazing variety of ways, ranging from solid-like when the shaking is weak, to fluid-like and even gas-like behavior at strong shaking [Jaeger *et al.* 1996; Aranson and Tsimring 2006]. In this lecture we bring some order in this dynamical labyrinth by constructing a *phase diagram* for vertically shaken granular matter in a 2D container [Eshuis *et al.* 2007].

At weak shaking, the observed phenomena (a bouncing bed, which starts to exhibit standing waves like a violin string when the shaking is increased) are well described by mechanical models [Sano 2005; Eshuis *et al.* 2007].

At stronger shaking, the phenomena are better described by granular hydrodynamics [Goldhirsch 2003]. For increasing shaking strength, the standing waves first give way to the Leidenfrost state (in which a dense cloud of particles is held afloat by a gas-like layer of fast particles underneath [Eshuis *et al.* 2005]), followed by a transition to buoyancy driven convection: The particles now form counter-rotating rolls very similar to Rayleigh-Bénard convection rolls in an ordinary fluid [Khain and Meerson 2003; Eshuis *et al.* 2008]. Finally, at even higher shaking strengths the grains are swept into a gaseous state, moving wildly throughout the container. We will focus especially on the hydrodynamic description of the phenomena at strong shaking.

### References

- Aranson I S and Tsimring L S 2006, *Rev. Mod. Phys.* **78**, 641  
Eshuis P, van der Weele K, van der Meer D, and Lohse D 2005, *Phys. Rev. Lett.* **95**, 258001.  
Eshuis P, van der Weele K, van der Meer D, Bos R, Lohse D 2007, *Phys. Fluids* **19**, 123301.  
Eshuis P, Alam M, van der Meer D, van der Weele K, Luding S, and Lohse D 2008, *Buoyancy-driven convection in vibrated granular matter: Experiment, numerics, and theory*, preprint.  
Goldhirsch I 2003, *Annu. Rev. Fluid Mech.* **35**, 267  
Jaeger H M, Nagel S R, and Behringer R P 1996, *Rev. Mod. Phys.* **68**, 1259.  
Khain E and Meerson B 2003, *Phys. Rev. E* **67**, 021306.  
Sano O 2005, *Phys. Rev. E* **72**, 051302.

---

# Granular Matter Dynamics

## II. Faraday's heaping effect unravelled

Ko van der Weele

*Mathematics Department,  
University of Patras,  
26500 Patras, Greece*

One of the classic and most enigmatic phenomena exhibited by granular matter is Faraday heaping: When a bed of fine sand is vertically vibrated, its initially flat surface breaks into a landscape of heaps, which in the course of time tend to coarsen into a single larger heap [Faraday 1831; Behringer *et al.* 2002]. As was already suggested by Faraday, the *surrounding air* must play a crucial role in this process since at very low pressures, when the air drag on the particles can be ignored, no heaping of any kind is observed [Pak *et al.* 1995].

Until recently, the heaping had been studied mostly experimentally, giving rise to several rivaling explanations: internal avalanches [Laroche *et al.* 1989], horizontal pressure gradients [Thomas and Squires 1998], and enhanced stability of inclined surfaces [Duran 2002]. Here we describe numerical simulations, closely corresponding to experiment, that confirm the horizontal pressure mechanism of Thomas and Squires. We show that the heaps are formed by the air that flows - through the bed - into the void beneath the bed when this detaches from the vibrating bottom. Our simulations also explain the eventual coarsening into one single heap [Van Gerner *et al.* 2007].

### References

- Behringer R P, van Doorn E, Hartley R R, and Pak H K 2002, *Granular Matter* **4**, 9.  
Duran J 2002, *C. R Physique* **3**, 217.  
Faraday M 1831, *Philos. Trans. R. Soc. London* **121**, 299.  
Laroche C, Douady S, and Fauve S 1989, *J. Phys.* **50**, 699.  
Pak H K, van Doorn E, and Behringer R P 1995, *Phys. Rev. Lett.* **74**, 4643.  
Thomas B and Squires A M 1998, *Phys. Rev. Lett.* **81**, 574.  
Van Gerner HJ, van der Hoef M, van der Meer D, and van der Weele K 2007, *Phys. Rev. E* **76**, 051305.

# Chaotic scattering in the regime of weakly overlapping resonances

Hans A. Weidenmüller

*Max-Planck-Institute for Nuclear Physics,  
Heidelberg, Germany*

Chaotic quantum scattering occurs when Schrödinger waves are scattered by a system with chaotic classical dynamics. The eigenvalues of the system manifest themselves as resonances with average spacing  $D$  and average width  $\Gamma$ . For chaotic systems, the spectral fluctuation properties of these eigenvalues coincide with the predictions of random-matrix theory (RMT) for matrices of the same symmetry class (“Bohigas–Giannoni–Schmit conjecture”). For time-reversal invariant systems, the matrix ensemble is the Gaussian Orthogonal Ensemble (GOE) of real and symmetric matrices.

The theory of chaotic scattering uses RMT or related models to predict average cross sections and correlation functions of scattering amplitudes in all three regimes, the regime of isolated resonances ( $\Gamma \ll D$ ), the regime of weakly overlapping resonances ( $\Gamma \approx D$ ), and the regime of strongly overlapping resonances ( $\Gamma \gg D$ ) (the “Ericson regime”). After reviewing some results for  $\Gamma \ll D$  and for  $\Gamma \gg D$  (where the theory has been thoroughly tested), I will focus attention on the comparison of recent experimental results on microwave billiards of the Darmstadt group with theory in the regime  $\Gamma \approx D$ .

---

# Preponderance of ground states with spin zero and/or positive parity

Hans A. Weidenmüller

*Max-Planck-Institute for Nuclear Physics,  
Heidelberg, Germany*

To explore generic features of the nuclear shell model, one often uses a random-matrix model, the two-body random ensemble (TBRE). In the TBRE, the two-body matrix elements of the shell model are taken to be Gaussian-distributed random variables with mean values zero and a common second moment. The single-particle states within a major shell are assumed to be degenerate. In 1998, Johnson, Bertsch, and Dean observed that in the TBRE, ground states with spin zero occur much more frequently than corresponds to their statistical weight. In 2004, Zhao *et al.* showed that in the TBRE, states with positive parity are likewise favoured ground states. That “predominance” of ground states with fixed quantum numbers occurs also in atoms and for bosons and has, therefore, found wide attention.

In the lecture, I will present these surprising facts and the explanations that have been put forward in the literature. I will then focus attention on a model which permits a detailed analytical and numerical study of the preponderance of ground states with positive parity. The model, worked out in collaboration with Thomas Papenbrock, leads to a thorough understanding of the phenomenon.

# Spectral Fluctuation Properties of Constrained Ensembles of Random Matrices

Hans A. Weidenmüller

*Max-Planck-Institute for Nuclear Physics,  
Heidelberg, Germany*

Wigner's random matrices enjoy wide applications in many fields of physics. Depending on the symmetries of the system, one uses one of Dyson's three canonical random-matrix ensembles to model successfully spectral fluctuation properties of or chaotic scattering on the system. That success of canonical random-matrix theory (RMT) is somewhat surprising since typically, the structure of the Hamiltonian of the system differs from that of a Gaussian random matrix: It may, for instance, be sparse or banded.

More realistic random-matrix models that take into account such structural details lack the invariance properties of the canonical ensembles. This is why it has so far not been possible to derive the spectral fluctuation properties of these ensembles analytically. A recent approach to such ensembles views them as constrained ensembles (some matrix elements of the canonical ensembles are constrained to vanish).

In the lecture, I will introduce the three canonical ensembles of RMT. I will then motivate the use of constrained ensembles, and I will present analytical results on level repulsion and on spectral fluctuation properties that have recently been obtained in collaboration with Thomas Papenbrock and Zdenek Pluhar.

---

# Planet formation

Michael Wilkinson<sup>1</sup>, Bernhard Mehlig<sup>2</sup>

<sup>1</sup>*Open University, Milton Keynes, England*

<sup>2</sup>*Göteborg University, Gothenburg, Sweden*

## Lecture 1: Observational evidence and review of theories

The theoretical basis for understanding the formation of the planets in our own solar system is far from secure. Also, in the last decade several hundred planets have been discovered orbiting other stars (Butler *et al* 2006) and these discoveries have included some surprises. For example, there are numerous extra-solar planets with very eccentric orbits, and gas-giant planets which orbit close to their star.

The standard theories for planet formation (Safranov, 1969, Goldreich and Ward 1973) involve building up planets from dust grains which are suspended in the gas of the circumstellar nebula surrounding young stars. I will review the standard models for the growth of planets starting from the aggregation of dust particles, and describe some of the difficulties faced by these models. I shall describe some recent attempts to circumvent these difficulties by embellishments of the standard models (Johansen *et al*, 2007, Kretke and Lin, 2007). At the end of the lecture I will mention a new and radically different theory which we have proposed (Wilkinson and Mehlig, 2008).

### References

Butler R. P. *et al*, *Astrophysical J.*, **646**, 505, (2006).

Safranov V. S., *Evolutsiia Doplanetnogo Oblaka*, (1969) (English transl.: Evolution of the protoplanetary cloud and formation of the Earth and planets, NASA Tech. Transl. F-677, Jerusalem: Israel Sci. Transl., 1972).

Goldreich P. and Ward W. R., *Astrophys. J.*, **183**, 1051-61, (1973).

Johansen A., Oishi J. S., Low M-M. M., Klahr H., Henning T. and Youdin A., *Nature*, **448**, 1022-5, (2007).

Kretke, K. A. and Lin, D. N. C., *Astrophys. J.*, **664**, L55, (2007).

Wilkinson M and Mehlig B, arXiv:0802.4099 (astro-ph).

## Lecture 2: Collision rates and relative velocities of turbulent aerosols

The standard model for planet formation involves the aggregation of dust particles in a circumstellar accretion disc. It is surmised that the gas in the accretion disc must be turbulent in order to explain the relatively short lifetime of accretion disc. This makes it necessary to understand the dynamics of dust particles in a turbulent gas. I will review some basic properties of turbulent flows (Frisch, 1995), and describe some recent results on the relative velocities and collision rates of the particles (Wilkinson *et al*, 2006, Mehlig *et al*, 2007, Gustavsson *et al*, 2008).

I shall also describe how the properties of the gas in the circumstellar accretion disc can be estimated from a steady-state theory with very few input parameters, using a theory of Shakura and Sunyaev (1973).

The dust particles are bound by van der Waals and other weak electrostatic forces. I will describe some estimates for the collision speed at which aggregates of dust particles will be fragmented (Dominik and Tielens, 1997), and show that the relative speeds of colliding dust aggregates make them very vulnerable to being fragmented (Wilkinson *et al*, 2008). This is a strong indication that an alternative theory must be sought.

### References

- U. Frisch, *Turbulence*, Cambridge University Press, (1995).
- M. Wilkinson, B. Mehlig and V. Bezuglyy, *Phys. Rev. Lett.*, **97**, 048501, (2006)
- Mehlig B., Uski V. and Wilkinson M., *Phys. Fluids*, **19**, 098107, (2007).
- Gustavsson K., Mehlig B., Wilkinson M. and Uski V., arXiv:0802.4099 (physics.flu-dyn).
- Shakura N. I. and Sunyaev R. A., *Astronomy and Astrophysics*, **24**, 337, (1973).
- Dominik C. and Tielens A. G. G. M., *Astrophys. J.*, **480**, 647-73, (1997).
- Wilkinson M., Mehlig B. and Uski V., *Astrophysical J. Suppl.*, **176**, 484, (2008).



### Lecture 3: Can gravitational direct collapse create planets?

I shall describe an alternative hypothesis for the formation of planets, developed in collaboration with Bernhard Mehlig, which we term *Concurrent Collapse*. According to our hypothesis, when a cloud of interstellar gas collapses to form a star, it fragments, giving rise to smaller objects which are gravitationally bound to the star. These *juvenile planets* are initially formed in non-circular orbits and have an elemental composition which is representative of the star. The juvenile planets can interact with the accretion disc in such a way that their orbit and their composition can be dramatically changed. Collisions between juvenile planets are also possible. Our hypothesis avoids having to resolve the difficulties faced by the dust aggregation model. It also provides satisfying explanations for the existence of exoplanets with eccentric orbits, for the occurrence of FU Orionis outbursts and for the melting of *chondrules* found in meteorites. Our explanation will be contrasted with those offered in the framework of the standard models (Ford *et al* 2003, Lodato and Clark, 2004, Hewins, 1997).

The concurrent collapse hypothesis depends upon the assumption that a gravitationally collapsing gas cloud will fragment. Gravitational collapse is imperfectly understood process. I will describe Jeans' theory for determining the size of objects formed by gravitational collapse, and discuss some of the arguments which have been advanced in the past to support the view that collapse is normally accompanied by fragmentation (Hoyle 1953, Low and Lynden-Bell, 1976, Padoan and Nordlund, 2002). None of these are really satisfactory. I will present an outline for a new theoretical approach to this problem.

#### References

- Ford E. B., Rasio F. A. and Yu K., in *Scientific Frontiers in Research on Extrasolar Planets*, ASP Conference Series, **294**, eds. Deming D. and Seager S., ASP, San Francisco, p.181, (2003).  
Lodato G. and Clark C. J., *Mon. Not. R. astr. Soc.*, **353**, 841-52, (2004).  
Hewins R. H., *Ann. Rev. Earth & Planetary Sci.*, **25**, 61-83, (1997).  
Hoyle F., *Astrophysical J.*, **118**, 513, (1953).  
Low C. and Lynden-Bell D., *Mon. Not. R. astr. Soc.*, **176**, 367-90, (1976).  
Padoan P. and Nordlund Å., *Astrophysical J.*, **576**, 870-9, (2002).

# Bifurcations, order, and chaos in Bose-Einstein condensates with long-range interactions

Günter Wunner

*Institut für Theoretische Physik 1, Universität Stuttgart,  
70550 Stuttgart, Germany*

Bose-Einstein condensates with long-range atomic interactions, in addition to the contact interaction, open the possibility of studying the properties of degenerate quantum gases in which the relative strengths of long- and short-range interactions can be continuously adjusted by tuning the contact interaction via a Feshbach resonance. Examples are Bose-Einstein condensates of neutral atoms with electromagnetically induced attractive  $1/r$  interaction (O' Dell et al. 2000) and dipolar Bose-Einstein condensates (Santos et al. 2000). The achievement of Bose-Einstein condensation in a gas of chromium atoms (Griesmaier et al. 2005), with a large dipole moment, has opened the way to promising experiments on quantum gases with long-range interactions (Koch et al. 2008). In both types of condensates universal stability thresholds exist where collapse of the condensates sets in. We show that these thresholds in fact correspond to bifurcation points where always two solutions of the Gross-Pitaevskii equation disappear in a tangent bifurcation (Papadopoulos et al 2007), one dynamically stable and the other unstable. We point out that the thresholds also correspond to “exceptional points” (Cartarius et al 2008a, b), i.e. branching singularities of the Hamiltonian. We analyze the dynamics of excited condensate wave functions via Poincaré surfaces of section and find both regular and chaotic motion, corresponding to (quasi-) periodically oscillating and irregularly fluctuating condensates, respectively (Wagner et al 2008). Stable islands are found to persist up to energies well above the saddle point of the mean field energy, alongside with collapsing modes.

## References

- Cartarius H, Main J and Wunner G 2008a *Phys. Rev A* **77** 01361  
Cartarius H, Fabčić T, Main J and Wunner G 2008b, preprint arXiv:0803.1799  
Griesmaier A, Werner J, Hensler S, Stuhler J and Pfau T (2005) *Phys. Rev. Lett.* **94** 160401  
Koch T, Lahaye T, Metz J, Fröhlich B, Griesmaier A and Pfau T 2008 *Nature Physics* **4** 218  
O' Dell D, Giovanazzi S, Kurizki G and Akulin V M 2000 *Phys. Rev. Lett.* **84** 5687  
Papadopoulos I, Wagner P, Wunner G and Main J 2007 *Phys. Rev A* **76** 053604  
Santos L, Shlyapnikov G V, Zoller P and Lewenstein 2000 *Phys. Rev. Lett.* **85** 1791  
Wagner P, Cartarius H, Fabčić T, Main J and Wunner G 2008, preprint arXiv:0802.4055

# Cold atoms in optical lattices with disorder

Jakub Zakrzewski

*Marian Smoluchowski Institute of Physics and  
Mark Kac Complex Systems Research Center Jagiellonian University, Kraków, Poland*

Cold atoms in optical lattices form a wonderful toolbox (Jaksch and Zoller 2005) for creating novel matter phases realizing a “condensed matter theorist dream”. This comes from an unprecedented control of the parameters in cold atoms physics where both the depth of the optical lattice (changing laser intensity or detuning) as well as atom-atom interactions (via external magnetic field and Feshbach resonances) can be modified with great precision. This allows for observation of quantum phase transitions (Sachdev 2001) as exemplified for the Bose-Hubbard (BH) tight binding model (Fisher et al. 1989, Jaksch et al. 1998, Greiner et al. 2002). Novel phases can be developed by studying Fermi-Bose or Bose-Bose mixtures as well as for the so called spinor condensates (where atoms in a few Zeeman sublevels are trapped together in optical traps) - review of Lewenstein et al. (2007) is a valuable introduction to that subject.

For a quantum chaos community particularly interesting may be studies of disordered systems. Here cold atomic settings allow to introduce disorder in a controllable and repeatable way using optical potentials created by laser speckles or bichromatic lattices (Damski et al. 2003, Roth and Burnett 2003). This opened up exciting possibilities for a clear observation of Anderson localization for weakly interacting bosons. This effort, after preliminary, discouraging results (Clément et al. 2005, Fort et al. 2005, Schulte et al. 2005) has led to a spectacular success quite recently (Billy et al. 2008, Roati et al. 2008). For strongly interacting bosons in a deep optical lattice the existence of a novel phase called Bose-glass phase has been predicted (Giamarchi and Schulz 1988, Fisher et al. 1989). Recently, the first attempt to produce this phase with ultracold atoms in a bichromatic quasi-disordered optical lattice has been reported (Fallani et al. 2007). This experiment is analysed using Time-evolving block-decimation (TEBD) algorithm of Vidal (2003).

The work supported by Marie Curie ToK project COCOS (MTKD-CT-2004-517186).

## References

- Billy J et al. 2008 arXiv:0804.1621  
 Clément D et al. 2005 *Phys. Rev. Lett.* **95**, 170409  
 Fort C et al., *Phys. Rev. Lett.* **95**, 170410  
 Damski D, Zakrzewski J, Santos L, Zoller P, & Lewenstein M, *Phys. Rev. Lett.* **91**, 080403  
 Fallani L, Lye J E, Guarrera V, Fort C, and Inguscio M 2007 *Phys. Rev. Lett.* **98**, 130404  
 Fisher M P A, Weichman P B, Grinstein G, & Fisher D S 1989 *Phys. Rev. B* **40**, 546  
 Fort C et al., *Phys. Rev. Lett.* **95**, 170410  
 Giamarchi T. and Schulz H J 1988 *Phys. Rev. B* **37**, 325  
 Greiner M, Mandel O, Esslinger T, Hänsch T W, & Bloch I 2002 *Nature* **415**, 39  
 Jaksch D and Zoller P 2005 *Ann. Phys.* **315**, 52  
 Jaksch D, Bruder C, Cirac J I, Gardiner C W, & Zoller P, 1998 *Phys. Rev. Lett.* **81**, 3108  
 Lewenstein M, et al. 2007 *Adv. Phys.* **56**, 243  
 Roati G et al. 2008 arXiv:0804.2609  
 Roth R. and Burnett K 2003 *Phys. Rev. A* **68**, 023604  
 Sachdev S 2001 *Quantum Phase Transitions* (Cambridge: Cambridge Univ. Press)  
 Schulte T. et al., *Phys. Rev. Lett.* **95**, 170411  
 Vidal G 2003 *Phys. Rev. Lett.* **91**, 147902

# Sources of ultra-high energy cosmic rays

Danilo Zavrtanik

*Laboratory for Astroparticle Physics,  
University of Nova Gorica, Nova Gorica, Slovenia*

One of the most fascinating puzzles in particle astrophysics today is that of the origin and nature of the highest energy cosmic rays. These particles have energies orders of magnitude beyond even the future capabilities of any earthly particle accelerator. Such energies are so extreme that they could arise in only the most violent places in the universe.

The Pierre Auger Observatory is a major international effort to make precise, high statistics studies of the cosmic rays with energies above  $10^{19}$  eV. The Southern Observatory was constructed in Province of Mendoza, Argentina and is designed to work in a hybrid mode incorporating both a ground array of 1.600 particle detectors spread over  $3.000 \text{ km}^2$  with fluorescence telescopes placed on the boundary of the surface array.

First data collected by the P. Auger Observatory provide evidence for anisotropy in the arrival directions of the cosmic rays with highest energies, which are correlated with the positions of relatively nearby Active Galactic Nuclei.

## References

- P. AUGER COLLABORATION: Abraham J et al. 2004 *Nucl. Inst. Meth.* **A523** 50  
P. AUGER COLLABORATION: Abraham J et al. 2007 *Science* **318** 938

---

# Entanglement and random quantum states

Marko Žnidarič

*Department of Physics, Faculty of Mathematics and Physics,  
University of Ljubljana, Ljubljana, Slovenia*

Quantum entanglement is one of the properties that makes quantum theory “strange” and different from classical physics. While the concept of entanglement has been known from the very beginning of quantum theory, only relatively recently have experiments advanced to the point of enabling manipulation of individual quanta. What is more, it has been realized that the entanglement can be used to our advantage. It enables for powerful quantum protocols that are better than the best classical procedures. In the lecture we shall address two questions: (i) what is the entanglement of typical quantum states, and (ii) how it comes that there is apparently no entanglement in macroscopic world. To explain the latter we will use properties of typical quantum states, that is states drawn according to unitarily invariant Haar measure.

In the first part we shall quantify the entanglement of random quantum states by calculating the average values of Schmidt coefficients. We will also study protocols for generating random quantum states. Efficiency of the protocol can be expressed in terms of a gap of Markovian process. In the second part we will show how to use properties of random states to explain the lack of entanglement for macroscopic systems. There will be two issues, practicality of detection and the role played by generic initial conditions. First, we shall show that the detection of entanglement in random quantum states is very hard, in fact, it gets exponentially hard with the number of particles. Second, we shall show that if the initial condition is a generic separable state then time evolution with an arbitrary hamiltonian will after very short time result in a state having no entanglement between subsystems of few degrees of freedom.

## References

- M. Nielsen and I. Chuang 2001 *Quantum computation and quantum information*, Cambridge University Press
- M. Žnidarič 2007 *J. Phys. A: Math. Theor.* **40** F105-F111
- M. Žnidarič 2007 *Phys. Rev. A* **76** 012318
- M. Žnidarič, T. Prosen, G. Benenti and G. Casati 2007 *J. Phys. A: Math. Theor.* **40** 13787-13798
- M. Žnidarič 2008 *preprint arXiv:0805.0523*

# Random Matrices, Quantum Chaos and Open Quantum Systems

Karol Życzkowski

*Jagiellonian University, Cracow, Poland  
and Center for Theoretical Physics, Polish Academy of Science*

A link between properties of quantized chaotic systems and random matrices will be reviewed. Dynamics of a closed quantum system can be represented by a unitary evolution of a pure state in the Hilbert space. We consider the case of a finite dimensional Hilbert space  $\mathcal{H}_N$ . Statistical properties of periodically driven quantum chaotic systems can be described by one of three circular ensembles of random unitary matrices, which belong to  $U(N)$ . The symmetry properties of the quantum system determine which of three universality classes – orthogonal, unitary or symplectic should be used.

To describe the effect of a possible interaction of the system in question with an environment one needs to work with density operators, which are Hermitian, positive and normalized. Discrete time evolution of a density matrix can be represented by so-called quantum operation (completely positive, trace preserving map). We are going to review the canonical Kraus form of such an operation and its representation by the dynamical (Choi) matrix.

To describe dynamics of an chaotic quantum system, interacting with an environment we introduce an ensemble of *random operations* and discuss practical algorithms to generate them. We investigate spectral properties of the associated superoperator  $\Phi$ , which sends the set of quantum states of size  $N$  into itself, and state a quantum analogue of the Frobenius-Perron theorem concerning the spectrum of stochastic matrices.

We derive a general formula for the density of eigenvalues of  $\Phi$  and show that for large  $N$  they are described by the real Ginibre ensemble of random matrices. We analyze the size of the spectral gap, which implies that a generic state of the system converges exponentially to an invariant state.

## References

- Haake F 1991 *Quantum Signatures of Chaos* (Berlin, Springer)  
Bengtsson I and Życzkowski K 2006 *Geometry of Quantum States* (Cambridge, Cambridge University Press)  
Bruzda W, Valerio Cappellini V, Sommers H-J, Życzkowski K 2008 Random Quantum Operations, preprint arXiv:0804.2361



# List of Participants

- **Mr. Shigeru Ajisaka**  
Advanced Institute for Complex Systems and Department of Applied Physics  
School of Science and Engineerings, Waseda University  
3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555  
Japan  
Email: g00k0056@suou.waseda.jp
- **Mr. Akira Akaishi**  
Department of Physics, Tokyo Metropolitan University  
1-1 Minami-Ohsawa, Hachioji-shi, Tokyo  
Japan  
Email: akaisi-akira@ed.tmu.ac.jp
- **Dr. Takuma Akimoto**  
1-12-66-616 Shibakubocho Nishitokyo-shi  
Tokyo  
Japan  
Email: akimoto@aoni.waseda.jp
- **Mr. Konstantinos Andriopoulos**  
22, Gyzi street, GR-15452  
Athens  
Greece  
Email: kand@aegan.gr
- **Dr. Alireza Bahraminasab**  
35. tower court, Lancaster LA1 4XH  
U.K.  
Email: a.bahraminasab@gmail.com
- **Mr. Benjamin Batistić**  
CAMTP, Krekova 2  
SI-2000 Maribor  
Slovenia  
Email: benjamin.batistic@gmail.com
- **Mr. Vladyslav Bezuglyy**  
Faculty of Mathematics, Computing and Technology, The Open University,  
Walton Hall, Milton Keynes MK7 6AA  
U.K.  
Email: v.bezuglyy@open.ac.uk
- **Mrs. Eleni Christodoulidi**  
Pantokratos 43, 26225  
Patras  
Greece  
Email: mathelegr1@yahoo.com
- **Mr. Anton Čotar**  
Senožeče 102b  
6224 Senožeče  
Slovenia  
Email: prevodnik.toni@gmail.com
- **Mr. Kristian Gustavsson**  
Jägaregaten 10A-44, 417 01  
Göteborg  
Sweden  
Email: kristian.gustaffson@physics.gu.se



- 
- **Mr. Tomohiro Hasumi**  
3-4-1 55-N-309A Okubo, Shinjyuku-ku, 196-8555  
Tokyo  
Japan  
Email: t-hasumi.1981@toki.waseda.jp
  
  - **Dr. Martin Horvat**  
Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana  
Jadranska 19, SI-1000 Ljubljana  
Slovenia  
Email: martin.horvat@fmf.uni-lj.si
  
  - **Mrs. Zhibek Kadyrsizova**  
Flat 7, House 87A, Gaidar street, 05009  
Almaty  
Kazakhstan  
Email: kzhibek@mail.ru
  
  - **Mr. Georgios Kanellopoulos**  
Department of Mathematics,  
University of Patras,  
26500 Patras, Greece  
Email: giorgoskan@lycos.com
  
  - **Mr. David Kenwright**  
Physics Department  
Lancaster University  
Lancaster  
LA1 4YB, UK  
Email: d.kenwright@lancaster.ac.uk
  
  - **Mr. Stayros Koutsokeras**  
KWSTH Palama 84  
Patras  
Greece  
Email: kutsok@master.math.upatras.gr
  
  - **Mr. Milan Kutnjak**  
Smetanoal ul. 17  
2000 Maribor  
Slovenia  
Email: milan.kutnjak@uni-mb.si
  
  - **Mrs. Taòà Langrova**  
Maòakova 8, Ostrava  
Czech Republic  
t.langrova@seznam.cz
  
  - **Mr. Cristoph Lhotka**  
Tü rkenschanzstrasse 17  
1180 Vienna  
Austria  
Email: christoph.lhotka@univie.ac.at
  
  - **Mr. Domenico Lippolis**  
837 State street NW  
Atlanta  
GA 30332-0430  
USA  
Email: domenico@gatech.edu
  
  - **Mr. Hofmann Lorenz**  
Reiherstrasse 60/2

73434 Aalen  
German  
Email: [lorenz-hofmann@arcor.de](mailto:lorenz-hofmann@arcor.de)

- 
- **Dr. German A Luna-Acosta**  
IFUAP, APdo Postal J-48  
72570 Puebla  
Mexico  
Email: gluna@sirio.ifuap.buap.mx
  - **Dr. Aleksandra Maluckov**  
Sindjelicev Trg 22/3  
1800 Niš  
Serbia  
Email: maluckov@junis.ni.ac.yu
  - **Mr. Thomas Meier**  
Paul-Gossen-Str. 34  
D-91025 Erlangen, Bayern  
Germany  
Email: thomas.meier@sonnenkinder.org
  - **Dr. Matej Mencinger**  
Stubiska 3  
2342 Ruše  
Slovenia  
Email: matej.mencinger@uni-mb.si
  - **Dr. Jose Antonio Mendez-Bermudez**  
18 Sur y Av. San Claudio. CP 72570  
Puebla  
Mexico  
Email: jmendezb@venus.ifuap.buap.mx
  - **Mr. Takahito Mitsui**  
26-36-#201, Kamiochia 2-chome  
Shinjyuku-ku  
Tokyo  
Japan  
Email: t.mitsui@aoni.waseda.jp
  - **Mr. Kenji Orihashi**  
3-4-1 + 55N-310 Aizawa Laboratory  
169-8555 Okubo  
Shinjuku-ku  
Tokyo  
Japan  
Email: orihashi-1026@aoni.waseda.jp
  - **Dr. Rytis Paškauskas**  
Via F. Severo, 29  
34133 Trieste TS  
Italy  
Email: rytis.paskauskas@elettra.triste.it
  - **Mr. Iztok Pižorn**  
Andraz nad Polzelo 20  
3133 Polzela  
Slovenia  
Email: iztok.pizorn@fmf.uni-lj.si
  - **Mr. Soya Shinkai**  
5-3-15-104, Kyodo  
Setagaya-Ku  
156-0052, Tokyo  
Japan  
Email: soya@toki.waseda.jp

- **Dr. Timur Tudorovskiy**  
Renthof 5  
D-35032 Marburg  
Germany  
Email: timur.tudorovskiy@physik.uni-marburg.de
- **Mrs. Zhadra Tultebayeva**  
Flat 75, House 101, Raiymnek street  
050016 Almaty  
Kazakhstan  
Email: jadrulich@mail.ru
- **Dr. Gregor Veble**  
University of Nova Gorica  
Vipavska 13, P.P. 301  
Rožna Dolina  
SI-5000 Nova Gorica  
Slovenia  
Email: gregor.veble@p-ng.si
- **Mr. Gregor Vidmar**  
CAMTP, Krekova 2  
SI-2000 Maribor  
Slovenia  
Email: gregor.vidmar@uni-mb.si
- **Mr. Stanislav Vymetal**  
Na Grosi 27  
102 00 Prague  
Czech Republic  
Email: stanislav.vymetal@gemalto.com
- **Mr. Wei Wei**  
Max-Plank-Institute for Dynamics and Self-Organization  
Busenstr. 10  
37073 Goettingen  
Germany  
Email: wei@nld.ds.mpg.de
- **Mr. Wilhelm Pablo Karel Zapfe Zaldivar**  
Patricio Sanz 739  
03100 Ciudad de Mexico  
Distro Federal  
Mexico  
Email: wpkzz@yahoo.com.mx
- **Mr. Matija Žerdin**  
Bezenškova 42  
2000 Maribor  
Slovenia  
Email: matija\_zerdin@yahoo.com

---

# Abstracts of Short Reports

## Nonequilibrium Peierls Transition

**Shigeru Ajisaka and Shuichi Tasaki**

*Advanced Institute for Complex Systems and Department of Applied Physics, School of Science and  
Engineering Waseda University  
3-4-1 Okubo, Shinjuku-ku Tokyo 169-8555, Japan  
g00k0056@suou.waseda.jp*

To examine an effect of non-equilibrium steady state (NESS) on phase transition, we analyze one-dimensional conductor which shows Peierls transition. Our system is connected to two heat baths which have different temperatures and chemical potentials, and is described by the following Hamiltonian.

$$\begin{aligned}
 H &= H_S + V + H_B \\
 H_S &= - \sum_{n=-1}^L \left( t_{n+1,n} c_{n+1}^\dagger c_n + (\text{h.c.}) \right) + \frac{K}{2} \sum_{n=-1}^L (y_{n+1} - y_n)^2 + \frac{M}{2} \sum_{n=0}^L \dot{y}_n^2 \\
 V &= \int dk v_k \left( c_0^\dagger a_k + c_L^\dagger b_k + (\text{h.c.}) \right), \quad H_B = \int dk (\omega_k a_k^\dagger a_k + \mu_k b_k^\dagger b_k)
 \end{aligned}$$

By considering a continuous counterpart of the system in which grid interval goes to infinitesimal and taking a mean field average for the lattice displacement, we constructed NESS. Last year, we studied the half filled case, and the existence of two stable CDW states, and one unstable CDW state under fixed bias voltage source is presented.

One natural question arises from the result: is it possible to increase a number of stable states. In the case of a half filled band, lattice displacement has period two thanks to the Peierls theorem. Thus, ground states are double-degenerated in equilibrium. We expect that the existence of the two stable states in NESS is related to this degeneracy. To check this hypothesis, non-equilibrium Peierls transition for the quarter filled case is discussed by introducing an appropriate field.

## On the ergodic measure of the non-equilibrium non-stationary state

**Takuma Akimoto**

*Department of Applied Physics, Advanced School of Science and Engineering, Waseda University, Okubo  
3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan.*

Lévy statistics and Lamperti statistics, which is characterized by the generalized arc-sine distribution, have attracted attention in non-equilibrium physics. However, it is still an open and important problem to found the measure characterizing the non-equilibrium state. In the ergodic theory, it has been shown that the time average of some observation functions converges to the universal distributions, such as the delta, the Mittag-Leffler, the generalized arc-sine and the stable distribution: Let  $(X, \mathcal{B}, m, T)$  be the dynamical system, then

$$\Pr \left\{ \frac{1}{n} \sum_{k=0}^{n-1} f \circ T^k \leq x \right\} \rightarrow G(x) \quad \text{as } n \rightarrow \infty, \quad (22)$$

where  $G(x)$  is the universal distribution which is determined by the dynamical system and the function  $f$ . These results can lead to the foundation of the ergodic measure characterizing the non-equilibrium non-stationary state.

In this talk we propose the definition of the non-equilibrium non-stationary state on the basis of the macroscopic observable, which results from the time average of the microscopic observable  $f$  in the dynamical system, and some specific ergodic measure are discussed.

### References

- Bardou F., Bouchaud J. P., Aspect A. and Cohen-Tannoudji C., *Lévy Statistics and Laser Cooling*, Cambridge University Press, 2002.
- Lamperti J., 1958 *Trans. Amer. Math. Soc.* **88** 380.
- Brokmann X., et al, 2003 *Phys. Rev. Lett.* **90** 12061.
- Margolin G. and Barkai E., 2006 *J. Stat. Phys.* **122** 137.
- Manneville P., 1980 *Le Journal de Physique* **41** 1235.
- Birkhoff G. D., 1931 *Proc. Nat. Acad. Sci. USA*, **17** 656.
- Darling D. A. and Kac M., 1957 *Trans. Amer. Math. Soc.* **84** 444.
- Aaronson J., *An Introduction to Infinite Ergodic Theory*, American Mathematical Society, 1997.
- Thaler M., 1998 *Trans. Amer. Math. Soc.* **350** 4593.
- Thaler M., 2002 *Ergo. Th. & Dynam. Sys.* **22** 1289.
- Akimoto, T, *J. Stat. Phys.*, to appear.

# Dynamics and stability of equilibria of a duopoly model

**Kostis Andriopoulos and Tassos Bountis**

*Centre for Research and Applications of Nonlinear Systems and  
Department of Mathematics, University of Patras, Patras GR-26500, GREECE*

*Email: kand@aegean.gr; bountis@math.upatras.gr*

The theory of oligopolies is a particularly active area of research using applied mathematics to answer questions that arise in microeconomics. It basically studies the occurrence of equilibria and their stability in market models involving few firms and has a history that goes back to the work of Cournot in the 19th century. More recently, interest in this approach has been revived, owing to important advances in analogous studies of Nash equilibria in game theory. In this talk, we first attempt to highlight the basic ingredients of this theory for a concrete model involving two firms. Then, after reviewing earlier work on this model, we describe our modifications and improvements, presenting results that demonstrate the robustness of the approach of nonlinear dynamics in studying equilibria and their stability properties. On the other hand, plotting the profit functions resulting from our modified model we show that their behavior is more realistic than that of other models reported in the recent literature.

## **References**

Cournot AA 1838 *Recherches sur les Principes Mathématiques de la Théorie des Richesses* (Hachette, Paris)  
Matsumoto A and Szidarovszky F 2007 *Nonlinear Dynamics, Psychology and Life Sciences* **11** 367



# Quantitative description of interactions between $\delta$ and $\theta$ brain waves

A. Bahraminasab, A. Stefanovska, P. V. E. McClintock

*Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK*

Brain waves contain different time scales related to different physiological processes that can interact with each other. Understanding the interactions can yield important information about the function of the brain. In a related example using the phase dynamics approach, interactions between  $\delta$ -waves, extracted from the electroencephalographic (EEG) signals, and cardio-respiratory signals were demonstrated [1]. It was shown that non-linear dynamics and information theory can be used to identify different stages of anaesthesia and the effect of different anaesthetics.

However, in order to model neuronal activity, deeper insight is needed i.e. considering also information about amplitude dynamics. The multidimensional Fokker-Planck equation or, equivalently, the associated Langevin equation can be used to model dissipative dynamical systems under the influence of noise [2]. Therefore, we analyze  $\delta$ - and  $\theta$ -waves based on specific characteristics of the one- and two-dimensional Kramers-Moyal coefficients to obtain the deterministic and stochastic parts of their interaction.

Finally, we apply stability analysis to the deterministic parts [3]. The fixed points and their dynamical exponents clearly reveal different interactions between the  $\delta$ - and  $\theta$ -waves in deep and light anaesthesia. We expect that the approach can be generalized to all time scales of the brain waves.

## References

1. Musizza B., Stefanovska A., McClintock P. V. E., Paluš M., Petrovčič J., Ribarič S., and Bajrović F. F. (2007), *J. Physiol.* 580, 315.
2. Haken H. 1983 *Synergetics*, Springer, Berlin.
3. Bahraminasab A., Ghasemi F., Stefanovska A., McClintock P. V. E., Friedrich R., In preparation.

## Fingerprints of Random Flows

Vlad Bezuglyy<sup>1</sup>, Michael Wilkinson<sup>1</sup> and Bernhard Mehlig<sup>2</sup>

<sup>1</sup>*Faculty of Mathematics, Computing and Technology The Open University, Walton Hall, Milton Keynes, MK7 6AA, England*

<sup>2</sup>*Department of Physics, Göteborg University, 41296 Gothenburg, Sweden*

We consider the patterns formed by small rod-like objects advected by a random flow in two dimensions. A simple theorem indicates that their direction field is non-singular. However, we show that singular behaviour can emerge in the long time limit. First, ‘flip lines’ emerge where the rods abruptly change direction by  $\pi$ . Later, these flip lines become so narrow that they disappear, but their ends remain as point singularities. These point singularities are of the same type as those seen in fingerprints.

## The Weibull - Log Weibull Transition in Interval-times of Earthquakes with Short Time Correlated Earthquakes Removed

Tomohiro Hasumi, Takuma Akimoto, and Yoji Aizawa

Department of Applied Physics, Advanced School of Science and Engineering, Waseda University, 169-8555 Tokyo, Japan

Analyzing seismic catalogs, for example the Japan Meteorological Agency Earthquake Catalog (JMA) and the Southern California Earthquake Catalog, and a two-dimensional spring-block model, we have revealed that the probability distributions of time intervals between successive earthquakes, inter-occurrence times, exhibit the transition from the Log Weibull regime to the Weibull one when the threshold of magnitude is increased. Based on the renewal theoretical analysis, we can conclude that the occurrence of earthquakes without correlated earthquakes is stationary because the mean and the second moment of the Weibull and the Log Weibull distribution are finite.

### References

- Hasumi T and Aizawa Y *Physics of Self-Organization Systems*, in press.  
 Hasumi T and Akimoto T, and Aizawa Y *submitted*.

## Triangle map and its ergodic properties

Martin Horvat

Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

The *triangle map* on the torus is a non-hyperbolic system featuring properties common to mostly chaotic systems such as diffusion, ergodicity and mixing. These non-obvious properties make this map interesting also in the quantum picture. Here we present some new views on the ergodicity and on the mixing in this system, which are yet not proved. The properties of the triangle system are studied by symbolically encoding the time evolution using two different schemes called *polygonal and binary description*. At a given time  $t$ , the points of the same code form disjoint partitions on the phase space. The number of these partitions, called *topological complexity*, grows with increasing time as  $O(t^3)$ . In general the properties of partitions scale in some way with  $t$ , which were closely examined. We also calculated the transition probabilities between partitions, referred to as the *Markov matrix of the map*, and study its spectral gap. The gap is shrinking with increasing time as  $O(t^p)$ ,  $p > 0$ , which is partially explained by the presented simple random model for the Markov matrix.

### References

- G Casati and T Prosen (1999) *Phys. Rev. Lett.* **83** 4729–32  
 G Casati and T Prosen (2000) *Phys. Rev. Lett.* **85** 4261–4264  
 M D Desposti, S O’Keefe and B Winn (2005) *Nonlinearity* **18** 073–1094  
 M. Horvat et. al. *Exploring ergodic and mixing properties of the triangle map* submitted to *Physica D*

## The effect of low frequency oscillations on cardiorespiratory synchronization

D. A. Kenwright, A. Bahraminasab, A. Stefanovska and P. V. E. McClintock

*Department of Physics, Lancaster University, UK*

The cardiac and respiratory systems behave in an oscillatory manner, and can be regarded as two weakly coupled, self-sustained oscillators. They have been shown to interact, to the extent that they can undergo episodes of synchronization (Schäfer *et al.* 1998, Lotrič & Stefanovska 2000, Pikovsky *et al.* 2001). Using non-linear dynamics, we can obtain further insight into different physiological states, such as stages of anaesthesia (Musizza *et al.* 2007). Here we look at how this interaction is affected by a perturbation in the form of physical exercise, by comparing the synchronization episodes of subjects when lying in a resting state with those during exercise on an exercise bike. We observe the changes to the oscillators that exercise causes by wavelet analysis, in particular an increase in modulation of the respiratory frequency, and how synchronization between the two oscillators is affected. It can be seen that the cardiorespiratory system undergoes transitions by switching between different close ratios of synchronization. Using a model of phase-coupled oscillators, we show how these transitions are mainly due to modulation by low frequency components of the oscillations (Kenwright *et al.* 2008).

### References

- Schäfer C, Rosenblum MG, Kurths J & Abel HH (1998) *Nature* **392** 239–240  
Lotrič MB & Stefanovska A (2000) *Physica A* **283** 451–461  
Pikovsky A, Rosenblum MG & Kurths J (2001). *Synchronization - a Universal Concept in Nonlinear Sciences*. Cambridge University Press, Cambridge  
Musizza B, Stefanovska A, McClintock PVE, Paluš M, Petrovčič J, Ribarič S, & Bajrović FF (2007) *J. Physiol.* **580** 315–326  
Kenwright DA, Bahraminasab A, Stefanovska A & McClintock PVE, (2008) *European Physiological Journal* (in press)

**ON DYNAMICS IN SOME DISCRETE QUADRATIC SYSTEMS IN THE PLANE USING  
THE ALGEBRAIC APPROACH**

**Milan Kutnjak<sup>1</sup>, Matej Mencinger<sup>2,3</sup>**

<sup>1</sup>*University of Maribor, Faculty of Electrical Engineering and Computer Science, Smetanova 17, 2000  
Maribor, Slovenia*

<sup>2</sup>*Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana, Slovenia*

<sup>3</sup>*University of Maribor, Faculty of Civil Engineering, Smetanova17, 2000 Maribor, Slovenia*

We consider the dynamics in some special cases of quadratic homogeneous discrete dynamical systems of the form

$$\begin{aligned} x_{k+1} &= a_1 x_k^2 + 2b_1 x_k y_k + c_1 y_k^2 \\ y_{k+1} &= a_2 x_k^2 + 2b_2 x_k y_k + c_2 y_k^2 \end{aligned} ; a_i, b_i, c_i \in \mathbb{R} \text{ for } i = 1, 2. \quad (23)$$

It is well known that homogeneous quadratic maps (23) are in one to one correspondence with two-dimensional commutative (nonassociative) algebras (Markus 1960):

*	$\vec{e}_1$	$\vec{e}_2$	}; $a_i, b_i, c_i \in \mathbb{R}$ for $i = 1, 2$ .
$\vec{e}_1$	$a_1 \vec{e}_1 + a_2 \vec{e}_2$	$b_1 \vec{e}_1 + b_2 \vec{e}_2$	
$\vec{e}_2$	$b_1 \vec{e}_1 + b_2 \vec{e}_2$	$c_1 \vec{e}_1 + c_2 \vec{e}_2$	

Algebraic concepts (such as structure of algebra and existence of special elements like idempotents and nilpotents) help us to study the dynamics of the corresponding discrete homogeneous quadratic maps. It is well-known that such systems can exhibit chaotic behavior (Kutnjak 2007). The simplest example is the complex-squaring map, which exhibits chaotic behavior on the unit circle which is the boundary,  $\partial B$ , of the set of all points with bounded forward orbits. We want to consider case-by-case (up to algebraic isomorphism) all commutative algebras in the plane in order to prove or disprove the existence of chaotic behavior in the corresponding discrete homogeneous quadratic system. Therefore, in this report we consider the sets  $\partial B$  (i.e. the generalized Julia sets) in some discrete homogeneous quadratic systems. We will describe the action of some structural algebraic properties and the existence of some special algebraic elements on the dynamics of the corresponding discrete homogeneous quadratic system. Some original results of the authors can be found in (Kutnjak and Mencinger 2008, Mencinger and Kutnjak).

### References

- Markus L 1960 *Ann. Math. Studies* **45** 185-213  
 Kutnjak M 2007 *Nonlinear Phenom. Complex Syst.* **10** 176-179  
 Kutnjak M and Mencinger M 2008 *Internat. J. Bifur. Chaos*, to appear  
 Mencinger M and Kutnjak M *Internat. J. Bifur. Chaos*, sent for publication

---

# Exponential Stability Estimates for Trojan Asteroids - Nekhoroshev Theorem meets Celestial Mechanics

Christoph Lhotka

*Institute for Astronomy,  
University of Vienna, Austria*

The concept of exponential stability in nonlinear dynamical systems can be traced back to 1955 (Moser) and Littlewood (1959). It was overshadowed for decades by the KAM theorem (Kolmogorov 1954, Arnold 1963, Moser 1962), which asserts stability for all times of those orbits, with initial conditions belonging to a Cantor set of tori of non-zero measure. In 1977 Nekhoroshev revived the research on it and analyzed in great detail the exponential stability times for general Hamiltonian systems near to integrable. Exponential (sometimes called practical) stability is of much greater interest from the physical point of view, as it can be applied to *all* orbits in open domains of the phase space, whether they lie on an invariant torus or not. The corresponding theorem proven by Nekhoroshev (1977) defines stability regions for a finite time  $T$  in both, regular and chaotic domains of the phase space. If the life-time of the physical system is shorter than the stability time derived from the Nekhoroshev estimates of the region, one can definitely say that orbits belonging to this region are stable from the practical point of view. This is the reason, why the Nekhoroshev theorem has to be considered at least as important as the KAM theorem as regards its relevance to the understanding of nonlinear dynamics. I will introduce the Nekhoroshev theory in short and show one typical application in Celestial Mechanics, where the mathematical theorem can reveal physical insights into the system, namely the motion near the 1:1 resonance of the elliptic restricted three body problem (Efthymiopoulos 2005, Lhotka et al. 2008).

Arnold V., 1963 *Russ. Math. Surv.* **18** 9-36.

Efthymiopoulos C., 2005 *Celest. Mech. Dyn. Astr.* **92** 29-52.

Kolmogorov A. N., 1954 *Russ. Math. Surveys* **18** 5.

Lhotka C., Efthymiopoulos, C., Dvorak R., 2008 *Month. Not. Roy. Astr. Society* **384** 1165-1177.

Littlewood, 1959 *Proc. London Math. Soc.(3)* **9** 343-372.

Moser J., 1973 *Princeton University Press* **77**

Nekhoroshev N., 1977 *Russ. Math. Survey* **32** 1-65.

## HOW WELL CAN ONE RESOLVE THE STATE SPACE OF A CHAOTIC FLOW?

**Domenico Lippolis and Predrag Cvitanović**

*Center for Nonlinear Science and School of Physics, Georgia Institute of Technology  
837 State street Atlanta -GA- USA*

All physical systems are affected by some noise that limits the resolution that can be attained in partitioning their state space. For chaotic, locally hyperbolic flows, this resolution depends on the interplay of the local stretching/contraction and the smearing due to noise. Our goal is to determine the finest possible partition of the state space for a given hyperbolic dynamical system and a given weak additive white noise of specified strength. We test these ideas on two models: the “skew Ulam” map and the Lozi attractor. We partition the state space by computing the local eigenfunctions of the Fokker-Planck evolution operator in the neighborhood of each periodic point, and use their widths to attain an optimal resolution of the state space. In both models the finest attainable partition for a given noise covers the state space by a finite tiling, and the Fokker-Planck evolution operator is represented by a finite Markov graph.

### References

- Cvitanović, Artuso, Mainieri, Tanner and Vattay, *Chaos: Classical and Quantum*, ChaosBook.org (Niels Bohr Institute, Copenhagen 2007)  
 Gaspard et al., *Physical Review E* **51**, 74 (1995)  
 H.H. Rugh, *Nonlinearity* **5**, 1237 (1992)

### Transport properties of waves and particles in periodic quasi-1D waveguides with mixed phase space

**G.A. Luna-Acosta, J.A. Méndez-Bermúdez, and J. Reyes Salgado**

*Instituto de Física, Universidad Autónoma de Puebla, Apdo. Postal J-48, Puebla, 72570, México*

We compute wave and ray transport quantities of a periodic quasi-one dimensional wave guide whose ray (or particle) dynamics undergoes the generic (KAM structure) transition from regular to global. We calculate the spatial diffusion  $\sigma_n^2(t)$  as a function of the time  $t$ , where  $n$  is the  $n^{\text{th}}$  cell of the periodic waveguide, as well as the evolution of the position distribution  $\rho(n)$  and momentum distribution  $\rho(p_x)$ . We find that  $\sigma_n^2(t) \sim t$  for global chaos and  $\sigma_n^2(t) \sim t^2$  for mixed chaotic dynamics with *unidirectional* direction. For mixed bi-directional motion  $\sigma_n^2(t) \sim t$  after a transient time which depends on the degree of chaoticity. We also solve the Helmholtz (Schroedinger) equation to obtain the energy band structure for infinitely long periodic waveguide and the conductance for finite waveguides for the three cases, global chaos, unidirectional mixed chaos and bidirectional mixed chaos. We analyze the wave transport properties in terms of the underlying classical dynamics.

---

**ON ALGEBRAIC APPROACH IN HOMOGENEOUS QUADRATIC SYSTEMS**

**Matej Mencinger**<sup>1,2</sup>, **Milan Kutnjak**<sup>3</sup>

<sup>1</sup> *Institute of Mathematics, Physics and Mechanics, Jadranska 19, 1000 Ljubljana, Slovenia*

<sup>2</sup> *University of Maribor, Faculty of Civil Engineering, Smetanova17, 2000 Maribor, Slovenia*

<sup>3</sup> *University of Maribor, Faculty of Electrical Engineering and Computer Science, Smetanova17, 2000 Maribor, Slovenia*

There is a one-to-one correspondence between homogeneous quadratic systems and nonassociative commutative finite dimensional real algebras (Markus, 1960). It is the right hand side of the system which allows us to introduce the algebraic concept into the dynamical system. The corresponding algebra multiplication  $*$  is uniquely defined by  $x * y = (Q(x + y) - Q(x) - Q(y)) / 2$ . Therefore, this kind of algebraic approach is applicable for the continuous systems,  $x' = Q(x)$ , as well as for the discrete systems  $x_{k+1} = Q(x_k)$ ;  $x \in \mathbb{R}^n$ . In this report we discuss some (dis)similarities between the continuous and discrete case. The origin is always a total degenerate critical point in the continuous case (c.f. Mencinger 2003) on one hand, and is (trivial) stable in the discrete case on the other hand. There is no chaotic behaviour in  $\mathbb{R}^2$  in the continuous case on one hand. On the other hand, it is well known that there is a chaotic behaviour in some discrete cases. We will also consider some (dis)similarities concerning the above described algebraic approach. In particular we will consider the influence of the algebraic structure (for example the existence of subalgebra or an ideal) to the corresponding continuous/discrete dynamical system. The meaning of algebra isomorphism is equal in both cases and it represents the basis for the linear equivalence classification of homogeneous quadratic systems. Next, we consider how the existence of some special algebraic elements (i.e. nilpotents of rank 2 and idempotents) reflects in the dynamics of the corresponding continuous/discrete system. Finally, we consider the existence of derivations and automorphisms in the corresponding algebra in order to obtain some special orbits in the system. By applying the algebraic approach to homogeneous quadratic systems the authors already solved some interesting problems in continuous systems (c.f. Mencinger, 2003, Mencinger and Zalar, 2005, Mencinger, 2006) as well as in discrete systems (c.f. Kutnjak, 2007, Kutnjak and Mencinger, 2008). Moreover, we are sure that the interplay between algebras and dynamical systems can create some new opportunities in both areas.

**References** Markus L 1960 *Ann. Math. Studies* **45** 185-213

Mencinger M 2003 *Nonlinearity* **16** 201-218

Mencinger M and Zalar B 2005 *Comm. Algebra* **33** 807-828

Mencinger M 2006 *Nonlinear Phenom. Complex Syst.* **9** 283-287

Kutnjak M 2007 *Nonlinear Phenom. Complex Syst.* **10** 176-179

Kutnjak M and Mencinger M 2008 *Internat. J. Bifur. Chaos*, to appear

## Emergence of criticality in surface corrugated waveguides

J. A. Mendez-Bermudez<sup>1</sup> and R. A. Aguilar-Sanchez<sup>2</sup>

<sup>1</sup>*Instituto de Física, Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico*

<sup>2</sup>*Facultad de Ciencias Químicas, Universidad Autónoma de Puebla, Puebla 72570, Mexico*

We investigate the statistical properties of eigenvalues and eigenvectors of surface corrugated waveguides depending on the degree of complexity of their boundaries. We focus on waveguide geometries whose phase space is ergodic in the classical limit. It is shown that the transition chaos-disorder,<sup>1</sup> driven by increasing the complexity of boundaries, is characterized by a transition in the statistical properties that goes from gaussian to critical. We model and explain this transition by the use of the Wigner-Lorenzian Random Matrix ensemble.<sup>2</sup> We also define a class of quantized chaotic deterministic systems that may show the emergence of criticality. This result is expected to be related to the new intermediate type of quantum chaos introduced recently in Ref. [3].

### References

<sup>1</sup>Mendez-Bermudez JA, Luna-Acosta GA, and Izrailev FM 2004 *Physica E* **22** 881

<sup>2</sup>Mendez-Bermudez JA, Kottos T, and Cohen D 2006 *Phys. Rev. E* **73** 036204

<sup>3</sup>Garcia-Garcia A and Wang J 2006 *Phys. Rev. E* **73** 036210

## Nonchaotic Stagnant Motion in a Marginal Quasiperiodic Gradient System

Takahito Mitsui

*Department of Applied Physics, Faculty of Science and Engineering, Waseda University, Tokyo 169-8555, Japan*

We present a one-dimensional dynamical system with a marginal quasiperiodic gradient, as a mathematical extension of the nonuniform oscillator [1],

$$\dot{x} = 1 - \frac{1}{2} \cos(2\pi x) - \frac{1}{2} \cos(2\pi kx),$$

which could be implemented in a multi-junction asymmetric SQUID modeled after the 3JJ SQUID ratchet proposed by Zapata et al. [2]. The system exhibits a nonchaotic stagnant motion, which is reminiscent of intermittent chaos. In fact, the density function of residence times near stagnation points obeys an inverse-power law due to a similar mechanism to type-I intermittency. However, contrary to the intermittent chaos, the alternation between long stagnant phases and rapid moving phases occurs not randomly but in a quasiperiodic manner. Particularly in the case of gradient with the golden ratio, the renewal of the largest residence time occurs on the positions corresponding to the Fibonacci sequence. Finally, the asymptotic long-time behavior in the form of nested logarithm is theoretically derived. In comparison with the Pomeau-Manneville intermittency, a significant difference in the relaxation property of the long-time average of dynamical variable is elucidated.

### References

[1] Strogatz S.H. 1994 *Nonlinear Dynamics and Chaos: with Applications in Physics, Biology, Chemistry, and Engineering* (Addison-Wesley, Reading, MA)

[2] Zapata I, Bartussek R, Sols F, and Hänggi P 1996 *Phys. Rev. Lett.* **77** 2292



## Turbulence in Diffusion Replicator Equation

**Kenji Orihashi and Yoji Aizawa**

*Department of Applied physics, Waseda University,  
3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan*

Dynamical behaviors in the diffusion replicator equation of three species are numerically studied. We study the following replicator dynamics with diffusion,

$$\left\{ \begin{array}{l} \frac{\partial X_i}{\partial t} = X_i \left( \sum_{j=1}^3 g_{ij} X_j - \sum_{j=1}^3 \sum_{k=1}^3 g_{jk} X_j X_k \right) + D \frac{\partial^2}{\partial r^2} X_i \\ \sum_{i=1}^3 X_i(r, t) = 1 \text{ and } 0 \leq X_1(r, t), X_2(r, t), X_3(r, t) \leq 1, [0 \leq r \leq L] \end{array} \right. \quad (24)$$

where  $X_i = X_i(r, t)$  is the frequency of  $i$  species (or player) ( $i = 1, 2, 3$ ),  $G = \{g_{ij}\}$  the interaction matrix,  $r \in [0, L]$  the one dim. space with periodic boundary,  $L$  the system size, and  $D$  the diffusion coefficient. Our motivation is the followings; when two or more heteroclinic cycles interact, what kind of complex behaviors come out?

Firstly, the bifurcation diagram for a certain parameter setting is drawn. Then it is shown that the turbulence appears with the supercritical Hopf bifurcation of a stationary uniform solution and it disappears under a subcritical-type bifurcation. Secondly, the statistical property of the turbulence near the supercritical Hopf onset point is analyzed precisely. Further, the correlation lengths and correlation times obey some characteristic scaling laws.

### References

Hofbauer J. and Sigmund K. 2003 *Bull. Amer. Math. Soc.* **40** 479

## Approach towards equilibrium in systems with long-range interactions

**R. Paškauskas, G. De Ninno**

*Sincrotrone Trieste, AREA Science Park, S.S.14 KM 163.5, 34012 Basovizza Trieste, Italy*

Considering dynamics of many particle interacting systems, the teaching of statistical physics postulates that on the microscopic level the disorder dominates and phase-space mixing produces the most likely homogeneous final state.

In a system where particles interact over long ranges, such as in a model of a free-electron laser in Sincrotrone Trieste, our numerical investigations have provided evidence that particles tend to linger in long-lived coherent states and that the transition to the final state takes longer than expected.

How does an initial formation of particles form a coherent state and gradually migrates towards an asymptotic equilibrium? What are the mechanisms of slowing down the phase-space mixing?

We will present results of our ongoing investigation of these problems using methods of dynamical systems. The specific question we will address is how do equilibrium structures and their manifolds mold the path of density evolution and how does the dramatic story of disorder winning over structure unwind in a system, representing many particles interacting with an electromagnetic wave, the Bonifacio model of free-electron lasers.

**Ergodicity and Complexity in Infinite Measure Dynamical Systems**  
 — *Scaling Laws and the Strong Intermittency of the Log-Weibull Map* —

**Soya Shinkai and Yoji Aizawa**

*Department of Applied Physics, Faculty of Science and Engineering,  
 Waseda University, Tokyo 169-8555, Japan*

Intermittent behaviors have been studied by use of the infinite ergodic theory [Shinkai (2006), Akimoto]. The most striking point is that inevitable statistical properties appear in the non-stationary chaos, such that the distribution of normalized Lyapunov exponents converges to the Mittag-Leffler distribution [Aaronson, Shinkai(2007)]. In the same way, the distribution of the Lempel-Ziv complexity in infinite ergodic systems does, and the scaling laws of the Lempel-Ziv complexity are theoretically evaluated [Shinkai(2006)].

Here we study a class of one-dimensional maps with an infinite measure  $T$  called “*Log-Weibull Map with an Infinite Measure*”, which is an extended model of Thaler’s example [Thaler]. The map  $T$  is given by,

$$T(x) = \begin{cases} x + \frac{1}{2}f(2x) & \text{for } x \in I_0 = [0, 0.5), \\ x - \frac{1}{2}f(2 - 2x) & \text{for } x \in I_1 = [0.5, 1], \end{cases}$$

where the function  $f$  is defined as follows:

$$f(t) = t^{1+\beta} \exp(1 - t^{-\beta}), \quad t \in [0, 1], \quad 0 < \beta < 1.$$

We prove that orbits generated by the map  $T$  reveal the strongest non-stationarity such as  $f^{-2}$  spectral fluctuations and the residence time distribution  $P(m)$  in intervals  $I_0$  and  $I_1$  is the log-Weibull one,

$$P(m) \sim m^{-1}(\log m)^{-1-\frac{1}{\beta}}.$$

As the Darling-Kac-Aaronson theorem in infinite ergodic theory says [Darling-Kac, Aaronson], the normalized partial sum of  $L_+^1$  function  $\frac{S_N}{a(N)} = \frac{1}{a(N)} \sum_{i=0}^{N-1} g \circ T_s$  ( $g \in L_+^1$ ) is the random variable which obeys the Mittag-Leffler distribution as  $N \rightarrow \infty$ , where  $a(N)$  is regularly varying at  $\infty$  with index  $\alpha = 0$  theoretically. Moreover, we show numerical and/or phenomenal aspects of the strongest non-stationarity such as  $\alpha \rightarrow 0$  ( $N \rightarrow \infty$ )

## References

- Shinkai S and Aizawa Y 2006 *Prog. Theor. Phys.* **116** 503–515  
 Akimoto T to appear in *J. Stat. Phys.*  
 Aaronson S 1997 *An Introduction to Infinite Ergodic Theory* (Amer. Math. Soc.)  
 Shinkai S and Aizawa Y 2007 *J. Korea. Phys. Soc.* **50** 267–271  
 Thaler M 1983 *Isr. J. Math.* **46** 67–96  
 Darling D A and Kac M 1957 *Trans. Amer. Math. Soc.* **84** 444–458

## On the theory of cavities with point-like perturbations

Timur Tudorovskiy, Ruven Höhmann, Ulrich Kuhl and Hans-Jürgen Stöckmann

*FB Physik, Philipps-Universität, Renthof 5, D-35032 Marburg, Germany*

The theoretical interpretation of measurements of “wavefunctions” and spectra in electromagnetic cavities excited by antennas is considered. Assuming that the characteristic wavelength of the field inside the cavity is much larger than the radius of the antenna, we describe antennas as “point-like perturbations”. This approach strongly simplifies the problem reducing the whole information on the antenna to four effective constants. In the framework of this approach we overcame the divergency of series of the phenomenological scattering theory and justify assumptions lying at the heart of “wavefunction measurements”. This selfconsistent approach allowed us to go beyond the one-pole approximation, in particular, to treat the experiments with degenerate states. The central idea of the approach is to introduce “renormalized” Green function, which contains the information on boundary reflections and has no singularity inside the cavity.

### References

Tudorovskiy T, Höhmann R, Kuhl U and Stöckmann H-J arXiv:0803.3556v1 [cond-mat.mes-hall] 25 Mar 2008

## Expanded boundary integral method and chaotic time-reversal doublets in quantum billiards

Gregor Veble, Tomaž Prosen, Marko Robnik

*University of Nova Gorica*

*Vipavska 13, P.P. 301, Rožna Dolina, SI-5000 Nova Gorica, Slovenia*

*and*

*CAMTP - Center for Applied Mathematics and Theoretical Physics*

*University of Maribor, Krekova 2, SI-2000 Maribor, Slovenia*

*gregor.veble@p-ng.si*

We present the expanded boundary integral method for solving the planar Helmholtz problem, which combines the ideas of the boundary integral method and the scaling method and is applicable to arbitrary shapes. We apply the method to a chaotic billiard with unidirectional transport, where we demonstrate existence of doublets of chaotic eigenstates, which are quasi-degenerate due to time-reversal symmetry, and a very particular level spacing distribution that attains a chaotic Shnirelman peak at short energy ranges and exhibits GUE-like statistics for large energy ranges. We show that, as a consequence of such particular level statistics or algebraic tunneling between disjoint chaotic components connected by time-reversal operation, the system exhibits quantum current reversals.

### References

Veble G, Prosen T and Robnik M 2007 *New J. Phys.* **9** 1



---

# Abstracts of Posters

## CYCLE EXPANSIONS OF A TWO-DIMENSIONAL PIECEWISE LINEAR MAP

Akira Akaishi and Akira Shudo

*Department of Physics, Tokyo Metropolitan University, Minami-Ohsawa, Hachioji, Tokyo 192-0397, Japan*

We investigate algebraic correlation of a certain class of two-dimensional piecewise linear map. For this map, there is a rigorous proof showing that there exists a series of parameter values at which polygon-shaped regular components and chaotic components coexist in phase space and these boundaries are strictly sharp (no resonance islands). We here present a systematic method to enumerate all the unstable periodic orbits in chaotic components with their linear stability and apply the cycle expansion method to derive the power-law behavior which manifests itself in the survival probability.

### References

- Wojtkowski M 1981 *Comm. Math. Phys.* **80** 453  
 Artuso R, Aurell E and Cvitanović P 1990 *Nonlinearity* **3** 325

## On the ergodic measure of the non-equilibrium non-stationary state: Generalized arc-sine law and stable law

Takuma Akimoto

*Department of Applied Physics, Advanced School of Science and Engineering, Waseda University, Okubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan.*

Limit theorems for the time average of non- $L^1(m)$  function in an infinite measure dynamical system are studied. We present the generalized arc-sine law and stable law in the skew modified Bernoulli map modeling the on-off intermittent phenomena.

Furthermore, we study the ergodic measure characterizing the non-equilibrium non-stationary state on the basis of the macroscopic observable, which results from the time average of the microscopic observable  $f$  in the dynamical system. Finally, we show that the generalized arc-sine distribution can be one of the ergodic measures of the non-equilibrium non-stationary state.

### References

- Lamperti J., 1958 *Trans. Amer. Math. Soc.* **88** 380.  
 Brokmann X., et al, 2003 *Phys. Rev. Lett.* **90** 12061.  
 Margolin G. and Barkai E., 2006 *J. Stat. Phys.* **122** 137.  
 Manneville P., 1980 *Le Journal de Physique* **41** 1235.  
 Birkhoff G. D., 1931 *Proc. Nat. Acad. Sci. USA*, **17** 656.  
 Darling D. A. and Kac M., 1957 *Trans. Amer. Math. Soc.* **84** 444.  
 Aaronson J., *An Introduction to Infinite Ergodic Theory*, American Mathematical Society, 1997.  
 Thaler M., 1998 *Trans. Amer. Math. Soc.* **350** 4593.  
 Thaler M., 2002 *Ergo. Th. & Dynam. Sys.* **22** 1289.  
 Akimoto, T, *J. Stat. Phys.*, to appear.

# Directionality of coupling and synchronization between coupled oscillators

A. Bahraminasab, A. Stefanovska, P. V. E. McClintock

*Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK*

The dynamics of many natural and man-made systems, and correspondingly the signals derived from them, is highly complex. This is especially true of the cardiovascular system and the brain. During the recent years much effort has been devoted to quantitative characterisation of the dynamical properties of complex systems by applying different time series analysis techniques. Complex dynamics has often been considered to result from oscillatory processes. Characterisation of the directionality of the couplings between different components of the system [1] and mutual synchronization [2] assume particular importance.

In this work, we present a recently introduced directionality index [3] for time series. It is based on conditional mutual information of data generated from comparison of neighboring values. We discuss the efficiency of the method and show that it can distinguish between different kinds of coupling, i.e. between unidirectional and bidirectional coupling, as well as reveal and quantify any asymmetry in bidirectional coupling. The fact that there is no need for preprocessing or fine-tuning of parameters makes the method very simple, computationally fast and robust.

Further, we introduce a new [4], simple and powerful method of detecting synchronization in noisy bivariate data. It is based on the detection of phase restriction, revealed as plateaus in plots of synchronization time as a function of a threshold defining the domain of acceptance for the phase difference. Unlike earlier methods, the criteria for fixing the optimal threshold and windows of observation arise naturally, facilitating reliable detection of synchronous epochs.

The two techniques will be illustrated using both numerical data and real data obtained from the cardiovascular system.

## References

1. Paluš M. and Stefanovska A. 2003, *Phys. Rev. E* 67 055201.
2. Pikovsky A., Rosenblum M., and Kurths J., 2001 *Synchronization - A Universal Concept in Nonlinear Sciences* Cambridge University Press, Cambridge.
3. Bahraminasab A., Ghasemi F., Stefanovska A., McClintock P. V. E., Kantz H. 2008, *Phys. Rev. Lett.* 100, 084101.
4. Bahraminasab A., Sheppard L. W., Stefanovska A., McClintock P. V. E., In preparation.

## NERVE PULSE PROPAGATION IN A CHAIN OF FHN NONLINEAR OSCILLATORS

E. Christodoulidi\*, S. Anastassiou\*, J. P. van der Weele\* and T. Bountis\*

\*Center for Research and Applications of Nonlinear Systems (CRANS), Department of Mathematics, University of Patras, GR-26500, Patras, Greece.

A particularly useful and instructive model for the study of nerve pulse propagation is described by the well - known FitzHugh Nagumo (FHN) partial differential equations. In the absence of diffusion, the FHN system represents a single point - like neuron and is expressed in terms of two Ordinary Differential Equations (ODEs) for the membrane electric potential and the recovery (ion) current. In this work, we connect  $N$  such FHN oscillators in a one - directional way, using the same coupling constant  $\alpha$ . We then apply to the first ODE a periodic square wave of period  $T$ , amplitude  $A$  and duration  $\Delta T$ , sufficient to excite the first neuronal oscillator. We then investigate ranges of parameter values for which the excited action potential wave train is transmitted to the subsequent FHN oscillators of the chain with the same period  $T$ . We thus discover conditions under which the transmitted wave has a period approximately equal to  $2T$  or  $3T, \dots$ , or fails to be transmitted far enough. We then add diffusion and also solve the FHN partial differential equations numerically to explore the effect of external forcing on the propagation of spatially extended pulses, which resemble more closely the action potential waves one encounters in actual nerve pulse propagation and cardiac muscle fiber contractions.

### References

Murray J 1989 *Mathematical Biology*, Springer, Berlin

Mikhailov A S 1990 *Foundations of Synergetics I : Distributed Active Systems*, Springer, Berlin

Nicolis G and Prigogine I 1989 *Exploring Complexity*, W. H. Freeman, New York

Nicolis G 1997 *Introduction to Nonlinear Science*, Cambridge University Press

Bountis T 1995 *Open Syst. Inf. Dyn: Fundamental Concepts of Classical Chaos : Part I* 3(1), 23 - 95

Bountis T, Starmer C F and Bezerianos A 2000 *Progr. Theor. Phys. Suppl.: Solitary Pulses and Wave Front Formation in an Excitable Medium* 139 12-33

## USE OF NORMALIZED RADIAL BASIS FUNCTION IN HYDROLOGY

Anton Čotar

FGG and FMF, University of Ljubljana, Slovenia

We used normalized radial basis function (NRBF) for reconstruction of river Reka runoff. We have measured data of Reka monthly runoff for period 1952-2006. By using precipitation and temperature data from Trieste (Italy) for period 1851-2006 we calculated numbers, that we interpret as Reka runoff for period 1851-1951. For prediction process we used multidimensional normal distribution, whose standard deviation was optimized on data from period 1952-1990. That was the learning process. The verification was done on period 1991-2006. For coefficient of quality of prediction we get 0,85. With optimized multidimensional normal distribution we then calculated a 1851-1951 vector, that represents Reka runoff for the same period. The problems with minimal and maximal runoff are still present, but this was expected, because the method is conservative. We get to the conclusions that this method is very useful for practical applications. It is more physical than often used linear regression and machine learning methods, because it is based on principle of measurement errors and maximal entropy.



---

**VARIABLE-RANGE PROJECTION MODEL FOR TURBULENCE-DRIVEN COLLISIONS**

**Kristian Gustavsson**

*Department of Physics, Göteborg University, 41296 Gothenburg, Sweden*

We discuss the probability distribution of relative speed  $\Delta V$  of inertial particles suspended in a highly turbulent gas when the Stokes numbers, a dimensionless measure of their inertia, is large. We identify a mechanism giving rise to the distribution  $P(\Delta V) \sim \exp(-C|\Delta V|^{4/3})$  (for some constant  $C$ ). Our conclusions are supported by numerical simulations and the analytical solution of a model equation of motion. The results determine the rate of collisions between suspended particles. They are relevant to the hypothesised mechanism for formation of planets by aggregation of dust particles in circumstellar nebula.

**References**

Gustavsson K, Mehlig B, Wilkinson M and Uski V 2008 *Preprint* arXiv:0802.2710

**Local first integrals of a cubic system  
of differential equations**

**Zhibek Kadyrsizova**

*al-Farabi Kazakh National University,  
Faculty of Mechanics and Mathematics, Almaty, Kazakhstan*

We consider a polynomial system of differential equations of the form

$$\dot{x} = x + p(x, y), \quad \dot{y} = -3y + q(x, y), \quad (25)$$

where  $p(x, y)$  and  $q(x, y)$  are homogeneous polynomials of degree three. In this paper we look for the first integrals of the system using as the main tool the Darboux method. In (Kadyrsizova and Romanovski) the linearizability problem for system (1) has been studied. In many cases considered there the construction of linearizing transformations require the knowledge of first integrals. In (Hu et al) it has been proved that such integrals exist. In the present paper we find the explicit expressions for the integrals. It allows to obtain the explicit formulas for the corresponding linearizing transformations of (Kadyrsizova and Romanovski). To perform the study we have developed some algorithms (and have implemented them in MAPLE) for finding invariant curves and their cofactors. Using the invariant curves we have constructed the local first integrals of system (1).

**References**

Z. Kadyrsizova, V. G. Romanovski, Linearizability of 1:-3 resonant system with homogeneous cubic nonlinearities, Proceedings of the International Symposium on Symbolic and Algebraic Computation, Linz, Austria, 2008, in press.

Z. Hu, V.G. Romanovski, D.S. Shafer, 1 : -3 Resonant Centers of Certain Cubic Systems on  $C^2$ , *Computers and Mathematics with Applications*, in press, doi:10.1016/j.camwa.2008.04.009.

## Critical Flow and Pattern Formation of Granular Matter on a Conveyor Belt

Georgios Kanellopoulos and Ko van der Weele

*Mathematics Department,  
University of Patras,  
26500 Patras, Greece*

We study the flow of granular material on a conveyor belt consisting of  $K$  connected, vertically vibrated compartments. A steady inflow is applied to the top compartment and our goal is to describe the conditions that ensure a continuous flow all the way down to the  $K$ th compartment. In contrast to normal fluids, flowing granular matter has a tendency to form clusters (due to the inelasticity of the particle collisions [Goldhirsch and Zanetti, 1993]); when this happens the flow stops and the outflow from the  $K$ th compartment vanishes.

Given the dimensions of the conveyor belt and the vibration strength, we determine the critical value of the inflow beyond which cluster formation is inevitable. Fortunately, the clusters are announced in advance (already below the critical value of the inflow) by the appearance of a wavy density profile along the  $K$  compartments. The critical flow and the associated wavy profile are explained quantitatively in terms of a dynamical flux model [Eggers, 1999; Van der Weele *et al.*, 2001]. This same model enables us to formulate a method to greatly increase the critical value of the inflow, improving the capacity of the conveyor belt by a factor two or even more.

### References

Eggers J 1999, *Phys. Rev. Lett.* **83**, 5322.

Goldhirsch I and Zanetti G 1993, *Phys. Rev. Lett.* **70**, 1619.

van der Weele K, van der Meer D, Versluis M, and Lohse D 2001, *Europhys. Lett.* **53**, 328.

---

## Paced respiration - furthering the understanding of cardio-respiratory synchronization and modulation during anaesthesia

D. A. Kenwright and A. Stefanovska  
*Department of Physics, Lancaster University, UK*

The cardio-respiratory system is an example of two coupled nonlinear oscillators that occur in nature, and provide the possibility to learn about characteristic phenomena. Of these, two are known: synchronization and modulation.

It is possible to perturb this system and observe the changes; for instance exercise increases the frequencies of both oscillators and effectively destroys synchronization and reduces modulation (Kenwright *et al.* 2008). Another perturbation can be achieved with anaesthesia, whereby the variability of the oscillators is reduced and we observe an increase in synchronization and modulation, as well as transitions in the synchronization ratio depending on the stage of anaesthesia (Stefanovska *et al.* 2000). From this it has been hypothesized that synchronization analysis may provide a means of controlling the depth of anaesthesia (Musizza *et al.* 2007).

Often during anaesthesia, respiration must be assisted by some mechanical respirator. Because of this, the question arises as to how the cardiorespiratory interaction changes if one of the oscillators (in this case respiration) is fixed. To answer this question, we obtain measurements from volunteers who breathe in time with a metronome. This is carried out for 3 different frequencies, above, below and approximately equal to the natural breathing frequency, and we compare with spontaneous breathing. Here we present results of analysis obtained from wavelet analysis (Bračič & Stefanovska 1999) and recently developed algorithms for detecting synchronization and determining directionality (Bahraminasab *et al.* 2008).

This work is part of the Brain, Respiration And Cardiac Causalities In Anaesthesia (BRACCIA) project, a Europe-wide interdisciplinary project involving anaesthetists, physiologists, neuroscientists, biomedical and electrical engineers, information theorists and physicists, with the aim being to reduce the incidence of awareness during anaesthesia.

### References

- Kenwright DA, Bahraminasab A, Stefanovska A & McClintock PVE (2008) *European Physiological Journal* (in press)
- Stefanovska A, Haken H, McClintock PVE, Hožič M, Bajrović F & Ribarič S (2000) *Physical Review Letters* **85** 4831–4834
- Musizza B, Stefanovska A, McClintock PVE, Paluš M, Petrovčič J, Ribarič S & Bajrović F *Journal of Physiology* **580** 315–326
- Bračič M & Stefanovska A (1999) *Nonlinear Phenomena in Complex Systems* **2** 68–77
- Bahraminasab A, Ghasemi F, Stefanovska A, McClintock PVE & Kantz H (2008) *Physical Review Letters* **100** 084101 1–4
- BRACCIA website: <http://www.lancs.ac.uk/depts/physics/braccia/>

## Complex chaos in the conditional dynamics of qubits

T. Kiss<sup>(1)</sup>, I. Jex<sup>(2)</sup>, G. Alber<sup>(3)</sup>, T. Langrová<sup>(2)</sup> and S. Vymětal<sup>(2)</sup>

(1) *Research Institute for Solid State Physics and Optics, H-1525 Budapest, P. O. Box 49, Hungary*

(2) *Department of Physics, FJFI ČVUT, Břehová 7, 115 19 Praha 1 - Staré Město, Czech Republic*

(3) *Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*

**Abstract.** The presence of complex chaos in iterative application of selective dynamics on quantum systems is a novel form of quantum chaos with true sensitivity to initial conditions. We present results for an ensemble of single qubits demonstrating how an efficient purification process can be destroyed due to the appearance of chaotic oscillations. The Julia sets of the studied process show a complicated structure with shapes strongly varying in dependence on the parameters of the dynamic. Techniques for the study of pure states are extended to include the cases of mixed states and entangled states.

### References

1. Deutsch D. et al 1996 *Phys. Rev. Lett.* **77** 2818
2. Macchiavello C. 1998 *Phys. Rev. Lett. A* **246** 385
3. Bechmann H. - Pasquinucci et al. 1998 *Phys. Lett. A* **242** 198
4. Terno D. R. 1999 *Phys. Rev. A* **59** 3320
5. Alber G. et al 2001 *J. Phys. A: Math. Gen.* **34** 8821
6. Poincaré H. 1892 *Les Méthodes Nouvelles de la Mécanique Céleste* (Gauthier-Villars, Paris)
7. Giannoni M.J., Voros A., Zinn-Justin J. 1991 *Chaos and Quantum Physics, Proceedings of the Les Houches Lecture Series, Session 52* (North-Holland, Amsterdam)
8. Cvitanović P., Artuso R., Mainieri R., Tanner G. and Vattay G. 2005 *Chaos: Classical and Quantum* (ChaosBook.org (Niels Bohr Institute, Copenhagen))
9. Schack R. et al. 1995 *J. Phys. A: Math. Gen.* **28** 5401
10. Bhattacharya et al. 2000 *Phys. Rev. Lett.* **85** 4852
11. Scott A.J. and Milburn G. J. 2001 *Phys. Rev. A* **63** 042101
12. Carlo G.G. et al. 2005 *Phys. Rev. Lett.* **95** 164101
13. Habib S., Jacobs K. Shizume K. 2006 *Phys. Rev. Lett.* **96** 010403
14. Habib S. et al. *e-print quant-ph/0505085*
15. Kiss T., Jex I., Alber G., Vymetal S. 2006 *Phys. Rev. A* **74** 040301
16. Fatou P. 1906 *C. R. Acad. Sci. Paris* **143** 546
17. Milnor J.W. 2000 *Dynamics in One Complex Variable* (Vieweg)
18. Lloyd S. and Slotine J.J. 2001 *Phys. Rev. A* **62** 012307
19. Milnor J.W. 1993 *Exp. Math.* **2** 37

**Transmission properties of the positional disordered photonic Kronig-Penney model.  
Experiment and theory**

**G.A. Luna-Acosta, N.Makarov, F.M. Izrailev**

*Instituto de Fisica, Universidad Autónoma de Puebla, Apdo. Postal J-48, CP. 72570, Puebla, México,*

**U. Kuhl, and H-J. Stöckmann**

*Fachbereich Physik, Philipps Universität Marburg, Renthof 5, D-35031, Germany*

We study the phenomena of localization in the transmission of a single mode through a positional disordered array formed by two alternating dielectrics of dielectric constants  $n_1$  and  $n_2 \neq n_1$ . The disorder is realized by randomly varying the width of one of the two dielectrics and keeping constant the width of the other. In the absence of disorder, this system is a photonic counterpart of the quantum Kronig-Penney model (\*). Under any amount of disorder there is complete transparency at frequencies such that  $kd_2 = m\pi$ , where  $k(\nu)$  is the wave vector at frequency  $\nu$  and  $d_2$  is the width of dielectric of constant width. In the regime of weak uncorrelated disorder we derive an analytical formula for the inverse localization length  $L_{loc}^{-1}$  and the average of the logarithm of the transmission, which is in perfect agreement with straight-forward transfer matrix calculations and with the transmission data of a micro-wave experiment.

**References**

(\*)G.A. Luna-Acosta, H. Schanze, U. Kuhl, and H.-J. Stoeckmann, 2008 *New J. of Physics* **10** 043005

**ON MOVING DISCRETE LOCALIZED MODES IN NONLINEAR LATTICES**

**Aleksandra Maluckov<sup>1</sup>, Milutin Stepić<sup>2</sup>, Goran Gligorić<sup>2</sup>, Ljupčo Hadžievski<sup>2</sup>, and Boris Malomed<sup>3</sup>**

<sup>1</sup> *Faculty of Sciences and Mathematics, P.O.B. 224, 18001 Niš, Serbia*

<sup>2</sup> *Institute for Nuclear Sciences-Vinča, P.O.B. 522, 11001 Belgrade, Serbia*

<sup>3</sup> *Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel*

Dynamics of discrete localized modes in nonlinear lattice like systems has been attracted a huge research interest in the nonlinear physics. It is worth to mention nonlinear optics, Bose-Einstein condensation phenomena, biophysics, solid state physics etc. Our investigations in this field are based on the interpretation of the localized mode dynamics in diverse nonlinear optical lattices, or lattices with different types of nonlinearity: cubic, cubic-quintic, saturable, etc. In addition we shown that the same methods are applicable to the other nonlinear problem as the Bose-Einstein condensates in deep periodic potential. The special interest has been the intriguing possibility to manipulate motion of localized intrinsic structures of high power in all these media. The concern was to interpret this possibility with respect to the correlation between the effect of nonlinearity and discreteness. Two general approaches are numerically tested: the free energy concept and mapping analysis. Only the second one has been shown appropriate in all studies.

**References**

Hadžievski Lj, Maluckov A, Stepić M and Kip D 2004 *Phys. Rev. Lett.* **93** 033901

Maluckov A, Hadžievski Lj and Stepić M 2006 *Physica D* **216** 95

Maluckov A, Hadžievski Lj and Malomed B A 2007 *Phys. Rev. E* **76** 046605

## Nonchaotic Stagnant Motion in a Marginal Quasiperiodic Gradient System

**Takahito Mitsui**

*Department of Applied Physics, Faculty of Science and Engineering, Waseda University, Tokyo 169-8555, Japan*

We present a one-dimensional dynamical system with a marginal quasiperiodic gradient, as a mathematical extension of the nonuniform oscillator [1],

$$\dot{x} = 1 - \frac{1}{2} \cos(2\pi x) - \frac{1}{2} \cos(2\pi kx),$$

which could be implemented in a multi-junction asymmetric SQUID modeled after the 3JJ SQUID ratchet proposed by Zapata et al. [2]. The system exhibits a nonchaotic stagnant motion, which is reminiscent of intermittent chaos. In fact, the density function of residence times near stagnation points obeys an inverse-power law due to a similar mechanism to type-I intermittency. However, contrary to the intermittent chaos, the alternation between long stagnant phases and rapid moving phases occurs not randomly but in a quasiperiodic manner. Particularly in the case of gradient with the golden ratio, the renewal of the largest residence time occurs on the positions corresponding to the Fibonacci sequence. Finally, the asymptotic long-time behavior in the form of nested logarithm is theoretically derived. In comparison with the Pomeau-Manneville intermittency, a significant difference in the relaxation property of the long-time average of dynamical variable is elucidated.

### References

- [1] Strogatz S.H. 1994 *Nonlinear Dynamics and Chaos: with Applications in Physics, Biology, Chemistry, and Engineering* (Addison-Wesley, Reading, MA)  
 [2] Zapata I, Bartussek R, Sols F, and Hänggi P 1996 *Phys. Rev. Lett.* 77 2292

## Turbulence in Diffusion Replicator Equation

**Kenji Orihashi and Yoji Aizawa**

*Department of Applied physics, Waseda University,  
3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan*

Dynamical behaviors in the diffusion replicator equation of three species are numerically studied. We study the following replicator dynamics with diffusion,

$$\begin{cases} \frac{\partial X_i}{\partial t} = X_i \left( \sum_{j=1}^3 g_{ij} X_j - \sum_{j=1}^3 \sum_{k=1}^3 g_{jk} X_j X_k \right) + D \frac{\partial^2}{\partial r^2} X_i \\ \sum_{i=1}^3 X_i(r, t) = 1 \text{ and } 0 \leq X_1(r, t), X_2(r, t), X_3(r, t) \leq 1, \quad [0 \leq r \leq L] \end{cases} \quad (26)$$

where  $X_i = X_i(r, t)$  is the frequency of  $i$  species (or player) ( $i = 1, 2, 3$ ),  $G = \{g_{ij}\}$  the interaction matrix,  $r \in [0, L]$  the one dim. space with periodic boundary,  $L$  the system size, and  $D$  the diffusion coefficient. Our motivation is the followings; when two or more heteroclinic cycles interact, what kind of complex behaviors come out?

Firstly, the bifurcation diagram for a certain parameter setting is drawn. Then it is shown that the turbulence appears with the supercritical Hopf bifurcation of a stationary uniform solution and it disappears under a subcritical-type bifurcation. Secondly, the statistical property of the turbulence near the supercritical Hopf onset point is analyzed precisely. Further, the correlation lengths and correlation times obey some characteristic scaling laws.

### References

- Hofbauer J. and Sigmund K. 2003 *Bull. Amer. Math. Soc.* 40 479

## Dynamical bottlenecks to intramolecular energy flow

**R. Paškauskas, C. Chandre, T. Uzer**

*Sincrotrone Trieste, AREA Science Park, S. S. 14 KM 163.5, 34012 Basovizza TS, Italy*  
*Centre de Physique Théorique - CNRS, Luminy - Case 907, 13288 Marseille cedex 09, France*  
*Center for Nonlinear Sciences, School of Physics, Georgia Institute of Technology, Atlanta, Georgia*  
*30332-0430, U.S.A.*

Vibrational energy flows unevenly in molecules, repeatedly going back and forth between trapping and roaming. We identify bottlenecks between diffusive and chaotic behavior, and describe generic mechanisms of these transitions, taking the carbonyl sulphide molecule OCS as a case study. The bottlenecks are found to be lower-dimensional tori; their bifurcations and unstable manifolds govern the transition mechanisms.

### References

R. Paškauskas and C. Chandre and T. Uzer 2008 *Phys. Rev. Lett.* **100** (8) 083001(4)

## Two-body random spin ensemble: a new type of quantum phase transition

**Iztok Pižorn, Tomaž Prosen, and Thomas H. Seligman**

*University of Ljubljana, Faculty of Mathematics and Physics, Department of Physics, Jadranska 19, SI-1000 Ljubljana*  
*Centro Internacional de Ciencias, Apartado postal 6-101, C.P.62132 Cuernavaca, Morelos, Mexico*

Random matrix models were introduced about fifty years ago by Eugene Wigner. The scope of applications has increased over the years including various fields in physics, more recently also quantum information theory where the concept of individual qubits and their interactions becomes important. In quantum information theory, however, qubits are taken to be distinguishable and it is very pertinent to formulate and investigate two-body random ensembles (TBRE) for such a case.

In the present work we study properties of such spin TBRE's by analyzing a parametrization in terms of the group parameters and the remaining parameters associated with the "entangling" part of the interaction. Using symmetry arguments we propose an adequate definition for such ensembles in a very general framework in terms of independent Gaussian distributed variables.

In order to show the relevance of the new ensemble we address the simplest topology, namely the quantum chain with nearest neighbor interactions. We focus on the ensemble averaged structure of the ground state described by the entanglement measures and spin correlations and demonstrate the existence of an unusual quantum phase transition (QPT) which is triggered by breaking of time-reversal invariance.

### References

Pižorn I, Prosen T, Mossmann S, Seligman T H 2008 *New J. Phys.* **10** 023020.

## Ergodic Properties for the Log-Weibull Map with a Uniform Measure

Soya Shinkai and Yoji Aizawa

*Department of Applied Physics, Faculty of Science and Engineering,  
Waseda University, Tokyo 169-8555, Japan*

Recently, the study of one-dimensional maps is developed for the understanding of Hamiltonian systems [Miyaguchi, Pikovskii], where both the hyperbolic region and the non-hyperbolic one coexist with a uniform Lebesgue measure. Aizawa showed that a universal statistical law is theoretically derived from the Nekoroshev theorem [Aizawa]. The law is that the probability density for the pausing time around KAM tori obeys the log-Weibull distribution asymptotically.

According to their studies, we study a class of one-dimensional maps  $T$  given by,

$$T(x) = \begin{cases} x + (1-c)f\left(\frac{x}{c}\right) & \text{for } x \in I_0 = [0, c), \\ x - c + cf^{-1}\left(\frac{x-c}{1-c}\right) & \text{for } x \in I_1 = [c, 1]. \end{cases}$$

We can prove that a uniform measure is derived from the Frobenius-Perron equation. Here we consider the case of  $c = (2\beta + 1)/(2\beta + 2)$  and the function

$$f(t) = t^{1+\beta} \exp(1 - t^{-\beta}), \quad t \in [0, 1], \quad 0 < \beta < 1.$$

Then the map is called “*Log-Weibull Map with a Uniform Measure*”. (In the case  $f(t) = t^{\frac{\beta}{\beta-1}}$  ( $\beta > 1$ ), the map is consistent with Miyaguchi’s one.) In numerical calculations, the inverse function  $f^{-1}$  needs to be written explicitly by use of the “*Lambert W function*” which is defined as  $z = W(z)e^{W(z)}$  [Corless]. We derived the inverse function explicitly

$$f^{-1}(t) = \left\{ \frac{1+\beta}{\beta} W\left(\frac{\beta}{1+\beta} e^{\frac{\beta}{1+\beta}} t^{-\frac{\beta}{1+\beta}}\right) \right\}^{-\frac{1}{\beta}}.$$

Unlike the log-Weibull map with an infinite measure, in this map the residence time distribution  $P(m)$  in the interval  $I_0$  obeys the differentiated log-Weibull distribution,

$$P(m) \sim m^{-2} (\log m)^{-1-\frac{1}{\beta}} \left\{ 1 + \frac{1+\beta}{\beta} (\log m)^{-1} \right\}.$$

The power spectral density reveals the following form:  $S(\omega) \sim \omega^{-1} (-\log \omega)^{-\frac{1}{\beta}}$  ( $\omega \ll 1$ ), and the correlation function  $C(\tau) \sim (\log \tau)^{-\frac{1}{\beta}}$  ( $\tau \gg 1$ ), which are consistent with theoretical calculations. Finally, We briefly discuss the interrelation between the log-Weibull map  $T$  and the Hamiltonian chaos [Kikuchi].

### References

- Miyaguchi T and Aizawa Y 2007 *Phys. Rev. E* **75** 066201  
Pikovskiy A S 1991 *Phys. Rev. A* **43** 3146-3148  
Aizawa Y 1989 *Prog. Theor. Phys.* **81** 249-253  
Corless R M et al. 1996 *Adv. Comput. Math.* **5** 329-359  
Kikuchi Y and Aizawa Y 1990 *Prog. Theor. Phys.* **84** 563-567



## Effects of point-like perturbations in billiards

**Timur Tudorovskiy, Ruven Höhmann, Ulrich Kuhl and Hans-Jürgen Stöckmann**  
*FB Physik, Philipps-Universität, Renthof 5, D-35032 Marburg, Germany*

We consider experiments with microwave cavities perturbed by point-like couplings. Using the general approach we describe several types of experiments and discuss different effects of the perturbation. In the presented theory the central place belongs to the “renormalized” Green function.

### References

Tudorovskiy T, Höhmann R, Kuhl U and Stöckmann H-J arXiv:0803.3556v1 [cond-mat.mes-hall] 25 Mar 2008

## Complex chaos in the conditional dynamics of qubits

**T. Kiss<sup>(1)</sup>, I. Jex<sup>(2)</sup>, G. Alber<sup>(3)</sup>, T. Langrová<sup>(2)</sup> and S. Vymětal<sup>(2)</sup>**

(1) *Research Institute for Solid State Physics and Optics, H-1525 Budapest, P. O. Box 49, Hungary*

(2) *Department of Physics, FJFI ČVUT, Břehová 7, 115 19 Praha 1 - Staré Město, Czech Republic*

(3) *Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*

**Abstract.** The presence of complex chaos in iterative application of selective dynamics on quantum systems is a novel form of quantum chaos with true sensitivity to initial conditions. We present results for an ensemble of single qubits demonstrating how an efficient purification process can be destroyed due to the appearance of chaotic oscillations. The Julia sets of the studied process show a complicated structure with shapes strongly varying in dependence on the parameters of the dynamic. Techniques for the study of pure states are extended to include the cases of mixed states and entangled states.

**References** Deutsch D. et al 1996 *Phys. Rev. Lett.* **77** 2818

Macchiavello C. 1998 *Phys. Rev. Lett. A* **246** 385

Bechmann H. - Pasquinucci et al. 1998 *Phys. Lett. A* **242** 198

Terno D. R. 1999 *Phys. Rev. A* **59** 3320

Alber G. et al 2001 *J. Phys. A: Math. Gen.* **34** 8821

Poincaré H. 1892 *Les Méthodes Nouvelles de la Mécanique Céleste* (Gauthier-Villars, Paris)

Giannoni M.J., Voros A., Zinn-Justin J. 1991 *Chaos and Quantum Physics, Proceedings of the Les Houches Lecture Series, Session 52* (North-Holland, Amsterdam)

Cvitanović P., Artuso R., Mainieri R., Tanner G. and Vattay G. 2005 *Chaos: Classical and Quantum* (Chaos-Book.org (Niels Bohr Institute, Copenhagen))

Schack R. et al. 1995 *J. Phys. A: Math. Gen.* **28** 5401

Bhattacharya et al. 2000 *Phys. Rev. Lett.* **85** 4852

Scott A.J. and Milburn G. J. 2001 *Phys. Rev. A* **63** 042101

Carlo G.G. et al. 2005 *Phys. Rev. Lett.* **95** 164101

Habib S., Jacobs K. Shizume K. 2006 *Phys. Rev. Lett.* **96** 010403

Habib S. et al. *e-print quant-ph/0505085*

Kiss T., Jex I., Alber G., Vymetal S. 2006 *Phys. Rev. A* **74** 040301

Fatou P. 1906 *C. R. Acad. Sci. Paris* **143** 546

Milnor J.W. 2000 *Dynamics in One Complex Variable* (Vieweg)

Lloyd S. and Slotine J.J. 2001 *Phys. Rev. A* **62** 012307

Milnor J.W. 1993 *Exp. Math.* **2** 37

## An Analytical Tractable Model for Dynamic Action Potential Encoding in Spatially Extended Neurons

W. Wei<sup>1,2</sup>, F. Wolf<sup>1,2,3</sup>

<sup>1</sup> *MPI for Dynamics and Self-Organization,*

<sup>2</sup> *Bernstein Center for Computational Neuroscience,*

and <sup>3</sup> *Faculty of Physics, University of Goettingen, Goettingen, Germany*

In the cerebral cortex, the results of all neuronal operations performed at the single cell level are coded into sequences of action potentials (APs). In the living brain, cortical neurons are subject to an immense synaptic bombardment, resulting in large fluctuations of their membrane potential and in temporally irregular AP firing. Recently, the AP encoding under conditions of such synaptic bombardment has received much attention and has been analyzed extensively in single compartment neuronal models. Real neurons, however, are spatially extended systems and physiological studies indicate that the site of action potential initiation of cortical neurons is located in a relatively small neuronal process, the proximal part of the axon. The impact of this geometry on AP wave form and AP encoding is a matter of ongoing controversy. To elucidate the impact of axonal AP initiation, we here take the axon as a semi-infinite cable and calculate the transfer of voltage fluctuation in the axon using the Green's function method. In the framework of Gaussian neuron model we obtain the spike-triggered voltage and variance at the soma when a spike is triggered at axon. We find that the spike-triggered variance is very small compared with the experimentally observed variability of thresholds at the soma. We also study the linear response function for the dynamical firing rate when action potentials are elicited in the axon and a small sinusoidal current is injected at soma.

### References

Naundorf, B., Wolf, F. and Volgushev, M. 2006 *Nature* **440** 1060-1063 McCormick, D. A., Shu, Y. and Yu, Y. 2006 *Nature* **445** doi: nature05523 Naundorf, B., Wolf, F. and Volgushev, M. 2007 *Nature* **445** E2-E3 Kole, M. et. al. 2008 *Nat. Neurosci* doi: 10.1038/nn2040

### Topology of chaotic scattering in four effective dimensions.

Christoff Jung and Karel Zapfe

*Instituto de Ciencias Fisicas*

*UNAM, Av. Universidad s/n, Col. Chamilpa, Cuernavaca, Morelos, 62210, Mexico.*

We will present a generalization to situations with four effective degrees of freedom of the Smalle Horseshoe construction. The goal of the approach will be finding relevant topological characteristics of the chaotic saddle and the invariant manifolds on data that can be experimentally measured. The method is basically the one used for the two dimensional case: We will study the topology of the singularity subsets of the scattering functions and we will try to find a way to correlate these to the rainbow singularities of effective cross section data.

# Concerts

---

---

# CAMTP

---

---

let's face chaos through nonlinear dynamics, 2008

## Concert

**Urška Orešič, piano & vocal**

*Kazinska dvorana, SNG Maribor*

*Tuesday, 1. July 2008, 21:00*

## Program

I. del

**F.Chopin:** Nocturno C# min, op.posth.  
**F.Chopin:** Minute Waltzer, op. 64 no.2  
**F.Chopin:** Ballade G min, op. 23

II. del

**A.L.Weber/T.S.Elliot:** Memory (from "Cats")  
**U.Orešič:** Under the stars  
**May/Queen:** Who Wants To Live For Ever  
**A.L.Weber/T.Rice:** Don't Cry For Me Argentina (from "Evita")

III. del

**B.Howard:** Fly Me To The Moon (In other words)  
**J.Schwartzinger/J.H.Mercer:** I Remember You  
**C.Porter:** I've Got You Under My Skin  
**S.Chan/J.Van Heusen:** All The Way  
**D.Fields:** The Way You Look Tonight  
**M.Greyer:** What A Difference A Day Made



**Urška Orešič** (1981) began with very first musical education in her home town Maribor. While taking her first years of piano lessons at the primary Music School she began to compose her own musical ideas by improvising on the piano. She continued with studying the piano and theory at Musical High school in Maribor at Professor Milena Sever's piano class. In 2001 she accepted to Music Academy in Ljubljana where she was studying the composition and music theory with Professor Pavel Mihelčič. She successfully graduated in 2005 with writing the one act Opera "An Evening with Rafael". She took parts at different seminars and Summer schools in Europe and worked with professors F. Burt, L. Voigtlaender and G. Fekete. Now she is continuing her studying with writing her Masters degree of composition and studying the piano with Professor Andrej Jarc at Music Academy in Ljubljana.

Her musical opus includes solo, choir, chamber and orchestral works; she mostly likes vocal-instrumental music. Her works have been performed for over 40 times, she also had 2 solo evenings with only her works performed. Her mostly performed compositions are Hymn to the oldest vine for soprano and piano, Vitis vinifera for chamber ensemble, White lyrics for soprano, horn, violin and piano. Other important works are Sketches for strings, performed by Chamber Slovene Filharmonic orchestra and Camerata Labacenzis and Triton which was recorded for Slovene promoting music with Radio Philharmonic orchestra and conductor En Shao. Urška Orešič was a member of Zois scholarship and a student's Preseren Prize. As academic musician and the professor of the musical art she is working as a composer, chamber musician, occasional pianist and singer. Permanently she is working as a piano teacher and accompanist at Gornja Radgona Music School.

---

---

# CAMTP

---

---

let's face chaos through nonlinear dynamics, 2008

## Concert

### Usha's rain

Metod Banko - vocal  
Bojan Krhlanko - drums  
Herman Luka Gaise - contrabass  
Matjaž Ferk - guitar

*Kavarna Art, Hotel PIRAMIDA*

*Saturday, 5. July 2008, 20:00*

## Program

1. Zmaj / Dragon (Metod B.)
2. Usha / Usha (Bojan K.)
3. Živa meja / Hedge (Metod B.)
4. Maček / Cat (Metod B.)
5. Med drugim in tretjim / Between the 2nd and the 3rd (Metod B.)



### **Usha's Rain** (quartet)

A quartet of young, creative jazz musicians, who are inspired by ethno music, jazz, sounds and images of nature, poetry... Their compositions are light, minimalistic, full of improvisations and interactions among the members of the quartet.



### **Metod Banko** (vocal)

His first creative work started in the early youth, mostly inspired by the nature. After finishing primary school in violoncello, he started to explore vocal music and finally devoted himself to singing. From the classical music to jazz and ethno... "In the season, I play on the pear leaf - but in the winter, evergreen ivy is the best." For the present he's a student of the jazz music in Klagenfurt, he cooperates with several bands, among them are Ursula Ramoves, Fanti z jazbečeve grape, St. polh sekstet...



### **Bojan Krhlanko** (drums)

While music is his companion from the very first steps, after finishing high school, he decided to migrate to Austria, where he passes the entrance exam and became a regular student of the Klagenfurt conservatory. For the present he is a member of various ensembles with whom he creates and concertizes worldwide. They are: The Stroj, Footprints trio, Black Coffee, Passion for jazz, Menima, Swing for life. As a member of the band The Stroj he received numerous prestigious awards and recorded three CD-s.



### **Herman Luka Gaiser** (contrabass)

He is a student at the Klagenfurt Jazz conservatory, where he studies contrabass. He cooperates with various bands and projects: Black Coffee, Swing4life, Elastik, GMS Band (performances in USA), Es-sauera Project feat. Bajsa Arifovska (Slovenia/Macedonia), Peter Genetrix - musical performance for theater (festival in Slovakia). He also cooperates with Samo Šalamon, Wellblott, Barbara Gabrielle...



### **Matjaž Ferk** (guitar)

He performs and composes music for many jazz ensembles (Footprints trio, Crossperiment, Elodie...). He is a guitar student with professor Thomas Wallisch. Earlier he attended jazz section on the Music and ballet High School Ljubljana. He took advanced studies at various music seminars lead by: Reggie Workman, Jerry Bergonzi, Aaron Goldberg, Bruce Gertz, Klemens Marktl...

---

---

# CAMTP

---

---

let's face chaos through nonlinear dynamics, 2008

## Concert

**Barbara Novak, piano**

*Kavarna Art, Hotel PIRAMIDA*

*Wednesday, 9. July 2008, 20:00*

## Program

### Johanes Brahms

- Intermezzo op.116 no.4
- Intermezzo op.118 no.1
- Intermezzo op.118 no.2
- Intermezzo op.117 no.2
- Intermezzo op.117 no.3
- Intermezzo op.116 no.3





**Barbara Novak** was born in Maribor, where she finished both primary and secondary music school with prof. Elizabeta Brglez. During that period she participated in numerous competitions at home and abroad where she won several prestigious awards, among them are second place in Competition of young Slovenian musicians (1995) and first place in international competition in Geneva (1997). Together with Oksana Pečeny and Karmen Pečar as a piano trio, she was a winner of the National competition of chamber orchestras (1999). In the same year, the same trio, also won the eminent Charles Hennen competition in Heerlen, Netherlands. For her achievements and several performances across Slovenia so as for recordings on the Radio Maribor, she was awarded the dr. Roman Klasinc Medal. With the orchestra of secondary music and ballet school Maribor, she performed the 1st movement of the Sergej Rahmaninov Piano concert in c-minor and with the violinist Oksana

Pečeny and Maribor string orchestra she also performed the Felix Mendelssohn Bartholdy Concert for violin, piano and string orchestra. In the year 2006 she graduated with distinction in musical arts with prof. Andrej Jarc, performing Sergej Rahmaninov Rhapsody on a theme of Paganini with the SNG Maribor orchestra and Milivoj Šurbka as a conductor. During her studies she was a Zois scholarship recipient. In 2001 she received the Lindau prize and in 2005 the Yamaha scholarship. She regularly attends seminars and summer schools with prominent professors, among them are S. Gadjevič, V. Lobanov, K. Polyzoides, J. Perry, A. Serdar, J. Siirala, R. Kinka, V. Efsathiadou, K. - H. Kmmmerling, K. Bogino. In the school year 2006/2007 she was a student of prof. K. Bogin in Bergam. Now she is a postgraduate student at the Ljubljana Music Academy with prof. Dubravka Tomšič - Srebotnjak.

---

---

# CAMTP

---

---

let's face chaos through nonlinear dynamics, 2008

## Concert

**Nikolaj Sajko, violoncello**  
**Piero Malkoč, contrabass**

*Kazinska Dvorana, SNG Maribor*

*Tuesday, 8. July 2008, 21:00*

## Program

**Benedetto Marcello** (1686 - 1739): Sonata t. 6 v G-duru

Adagio  
Allegro  
Grave  
Vivace

**Benedetto Marcello** (1686 - 1739): Sonata in D-major

Adagio  
Allegro  
Largo maestoso  
Vivace

**Giovanni Battista Sammartini** (1698 - 1775): Sonata in G-major

Adagio non troppo  
Allegro con espressione  
Vivace

**Luigi Boccherini** (1743 - 1805): Sonata in G-major

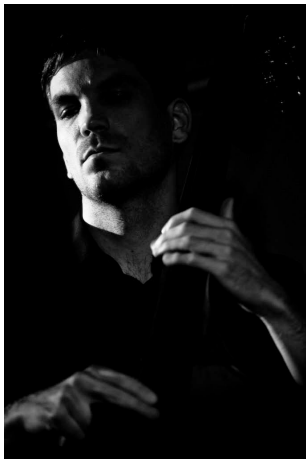
Largo  
Allegro alla Militaire Menuetto

**Luigi Boccherini** (1743 - 1805): Sonata in A-major

Allegro moderato  
Adagio cantabile  
Tempo di Minuetto



**Nikolaj Sajko** finished studies of cello performance at the Ljubljana Music Academy in the class of professor Ciril Škerjanec. Currently he is making his masters degree in performance at the Anton Bruckner University in Linz and working on his master thesis on theory of performance at the Ljubljana Music Academy. He is employed as co-principal cellist in the Symphony orchestra of Slovene national theater in Maribor. He has attended numerous master classes with professors I. Gavriš, E. Schoenfeld, A. Noras, C. Onczay and T. Khne. For his musical achievements he has received "Dr. Roman Klasinc" diploma and the "Antonio Tarsia" prize. He was a member of Gustav Mahler Youth Orchestra (artistic director Claudio Abbado) and is a member of Jeunesses Musicales World Orchestra. He has appeared as soloist with Maribor philharmonic orchestra, Gaudeamus chamber orchestra and Maribor youth orchestra. He is also devoting a lot of his time to chamber music.



**Piero Malkoč** finished secondary music school in Ljubljana as a pianist and two years later in double bass performance. He continued his studies of double bass performance at the Ljubljana Music Academy in the class of Zoran Markovič. He is member of numerous chamber orchestras - Solisti Piranesi, Savitra, Academia Maestro, Gaudeamus, with the later he appeared also as soloist. He is co-founder of the chamber orchestra Adrianis. In 2006 and 2007 he was a member of the Slovene Philharmonic Orchestra. Currently his is employed in the Symphony orchestra of Slovene national theater in Maribor.

## About composers

**Benedetto Marcello** (1686 -1739) was an Italian composer, writer, advocate and teacher. Born in Venice, Benedetto Marcello was a member of a noble family. Although he was a music student, his father wanted Benedetto to devote himself to law. He combined a life in law and public service with one in music. In 1711 he was appointed member of Venecian central government, and in 1730 he went to Pola as district governor. Benedetto Marcello composed a diversity of music including considerable church music, oratorios, hundreds of solo cantatas, duets, sonatas, concertos and sinfonias. Marcello was a younger contemporary of Antonio Vivaldi in Venice and his instrumental music enjoys a Vivaldian flavor. The sonatas in D major and G major were written and published in the years 1732 - 1734.

**Giovanni Battista Sammartini** (1698 -1775) was an Italian composer, organist, choirmaster and teacher. He counted Gluck among his students, and was highly regarded by younger composers including Johann Christian Bach. Sammartini is especially associated with the formation of the concert symphony through both the shift from a brief opera-overture style and the introduction of a new seriousness and use of thematic development that prefigure Haydn and Mozart. Sammartini's music is generally divided into three stylistic periods: the early period (1724-1739), which reflects a mixture of Baroque and Preclassical forms, the middle period (1740-1758), which suggests Preclassical form, and the late period (1759-1774), that displays Classical influences, including Mozart. Sammartini's middle period is regarded as his most significant and pioneering, during which his compositions in the gallant style of music foreshadow the Classical era to come. The sonata in G major was written and published in 1742.

**Luigi Rodolfo Boccherini** (1743 -1805) was a classical era composer and cellist from Italy, whose music retained a courtly and gallant style while he matured somewhat apart from the major European musical centers. Boccherini is most widely known for one particular minuet from his String Quintet in E and the Cello Concerto in B flat major. A virtuoso cellist of the first caliber, Boccherini often played violin repertoire on the cello, at pitch, a skill he developed by substituting for ailing violinists while touring. This supreme command of the instrument brought him much praise from his contemporaries, and is evident in the cello parts of his compositions. He wrote a large amount of chamber music, including over one hundred string quintets for two violins, viola and two cellos, a dozen guitar quintets, nearly a hundred string quartets, and a number of string trios and sonatas (including at least 19 for the cello). His orchestral music includes around 30 symphonies and 12 virtuoso cello concertos. Boccherini's style is characterized by the typical Rococo charm, lightness, and optimism, and exhibits much melodic and rhythmic invention. The sonatas in G major and A major were written and published around 1770.



---

---

# CAMTP

---

---

## Center for Applied Mathematics and Theoretical Physics

Univerza v Mariboru • University of Maribor

Krekova 2 • SI-2000 Maribor • Slovenia

Phone +(386) (2) 2355 350 and 2355 351 • Fax +(386) (2) 2355 360

e-mail [Robnik@uni-mb.si](mailto:Robnik@uni-mb.si) • <http://www.camtp.uni-mb.si>

Director: Prof. Dr. Marko Robnik

## The Supporting Scientific Institution of

The 7th International Summer School/Conference

### "Let's Face Chaos through Nonlinear Dynamics"

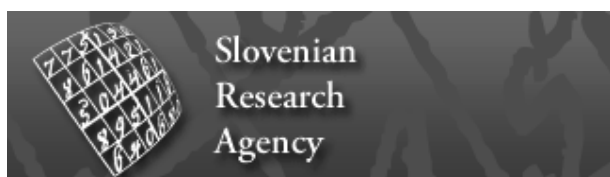
University of Maribor, Maribor, Slovenia

29 June - 13 July 2008

e-mail [chaos@uni-mb.si](mailto:chaos@uni-mb.si) • <http://www.uni-mb.si/chaos/2008/>

The mailing and contact address otherwise as for **CAMTP** above

thanks the Patron and the General Sponsor



**REPUBLIC OF SLOVENIA**

**MINISTRY OF HIGHER EDUCATION, SCIENCE AND  
TECHNOLOGY**

Trg Osvobodilne Fronte 13

1000 Ljubljana

SLOVENIA

# Main Sponsors

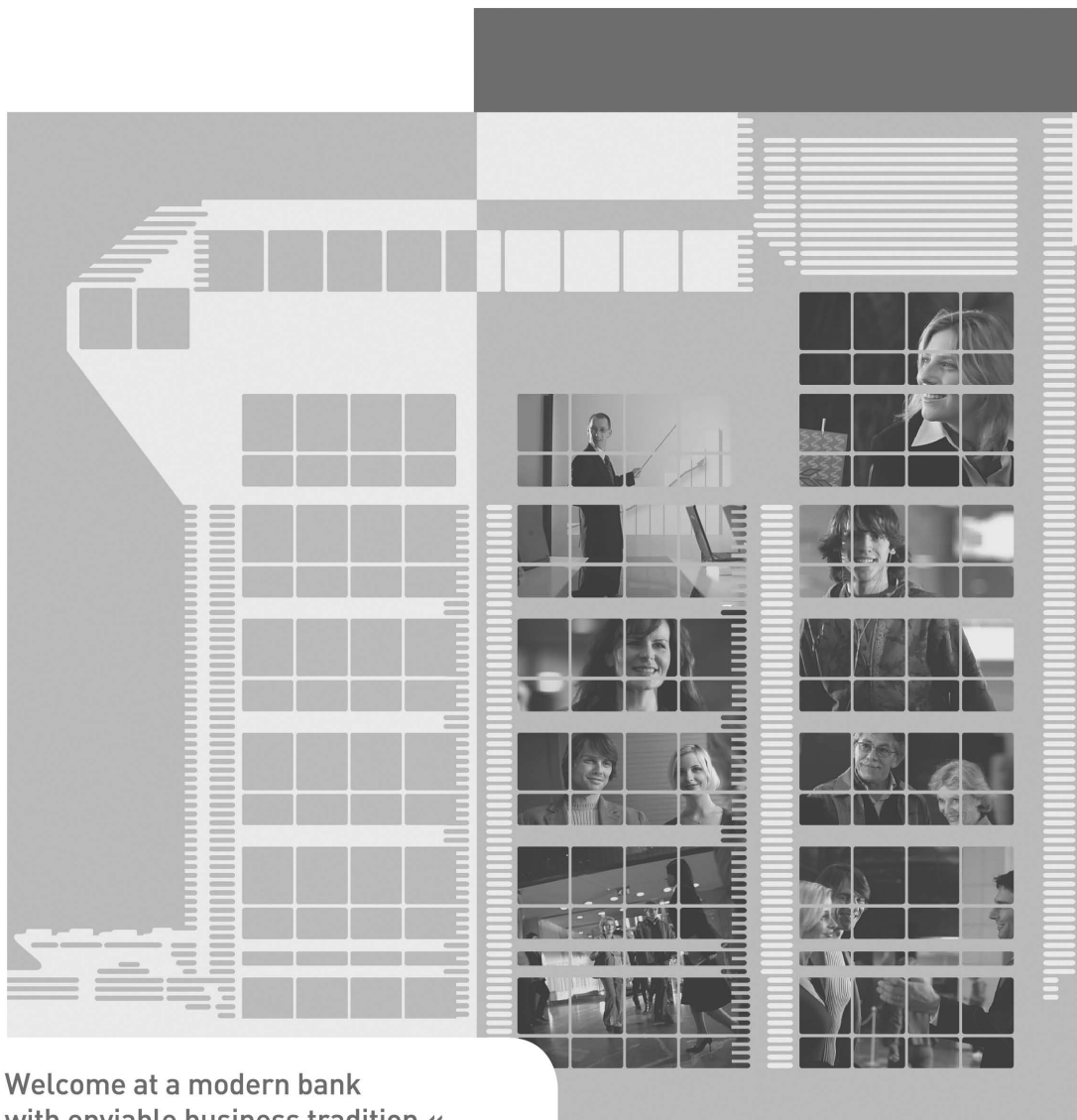
## Telekom Slovenije



## GEN Energija



and **Nova Kreditna Banka Maribor**



» Welcome at a modern bank  
with enviable business tradition.«

We **responsibly** grow together with the environment we work in. The foundation of our excellence is built on the **trust** of more than half a million people. With care and devotion we assure efficient solutions for our **clients, business partners** and **shareholders**. Being a pillar of a strong financial group we are able to respond to all the challenges the **future** brings.

[www.nkbm.si](http://www.nkbm.si)

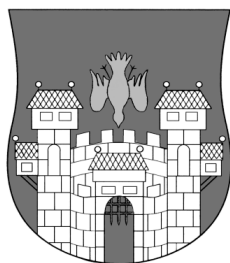
 Nova KBM



## Other Sponsors



Vinag Maribor



City of Maribor



Slovensko Narodno Gledališče Maribor



Lent Festival