Abstracts of Short Reports

Nonequilibrium Peierls Transition

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To examine an effect of non-equilibrium steady state (NESS) on phase transition, we analyze one-dimensional conductor which shows Peierls transition. Our system is connected to two heat baths which have different temperatures and chemical potentials, and is described by the following Hamiltonian.

$$\begin{aligned} H &= H_S + V + H_B \\ H_S &= -\sum_{n=-1}^{L} \left(t_{n+1,n} c_{n+1}^{\dagger} c_n + (\text{h.c.}) \right) + \frac{K}{2} \sum_{n=-1}^{L} (y_{n+1} - y_n)^2 + \frac{M}{2} \sum_{n=0}^{L} \dot{y}_n^2 \\ V &= \int dk\ v_k \left(c_0^{\dagger} a_k + c_L^{\dagger} b_k + (\text{h.c.}) \right), \quad H_B = \int dk\ (\omega_k a_k^{\dagger} a_k + \mu_k b_k^{\dagger} b_k) \end{aligned}$$

By considering a continuous counterpart of the system in which grid interval goes to infinitesimal and taking a mean field average for the lattice displacement, we constructed NESS. Last year, we studied the half filled case, and the existence of two stable CDW states, and one unstable CDW state under fixed bias voltage source is presented.
One natural question arises from the result: is it possible to increase a number of stable states. In the case of a half filled band, lattice displacement has period two thanks to the Peierls theorem. Thus, ground states are double-degenerated in equilibrium. We expect that the existence of the two stable states in NESS is related to this degeneracy. To check this hypothesis, non-equilibrium Peierls transition for the quarter filled case is discussed by introducing an appropriate field.

On the ergodic measure of the non-equilibrium non-stationary state

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Lévy statistics and Lamperti statistics, which is characterized by the generalized arc-sine distribution, have attracted attention in non-equilibrium physics. However, it is still an open and important problem to found the measure characterizing the non-equilibrium state. In the ergodic theory, it has been shown that the time average of some observation functions converges to the universal distributions, such as the delta, the Mittag-Leffler, the generalized arc-sine and the stable distribution: Let $(X, \mathcal{B}, m, T)$ be the dynamical system, then

$$\Pr\left\{\frac{1}{n}\sum_{k=0}^{n-1} f \circ T^k \leq x\right\} \to G(x) \quad \text{as } n \to \infty, \tag{1}$$

where $G(x)$ is the universal distribution which is determined by the dynamical system and the function f. These results can lead to the foundation of the ergodic measure characterizing the non-equilibrium non-stationary state.

In this talk we propose the definition of the non-equilibrium non-stationary state on the basis of the macroscopic observable, which results from the time average of the microscopic observable f in the dynamical system, and some specific ergodic measure are discussed.

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Dynamics and stability of equilibria of a duopoly model

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The theory of oligopolies is a particularly active area of research using applied mathematics to answer questions that arise in microeconomics. It basically studies the occurrence of equilibria and their stability in market models involving few firms and has a history that goes back to the work of Cournot in the 19th century. More recently, interest in this approach has been revived, owing to important advances in analogous studies of Nash equilibria in game theory. In this talk, we first attempt to highlight the basic ingredients of this theory for a concrete model involving two firms. Then, after reviewing earlier work on this model, we describe our modifications and improvements, presenting results that demonstrate the robustness of the approach of nonlinear dynamics in studying equilibria and their stability properties. On the other hand, plotting the profit functions resulting from our modified model we show that their behavior is more realistic than that of other models reported in the recent literature.

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Quantitative description of interactions between δ and θ brain waves

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Brain waves contain different time scales related to different physiological processes that can interact with each other. Understanding the interactions can yield important information about the function of the brain. In a related example using the phase dynamics approach, interactions between δ-waves, extracted from the electroencephalographic (EEG) signals, and cardio-respiratory signals were demonstrated [1]. It was shown that non-linear dynamics and information theory can be used to identify different stages of anæsthesia and the effect of different anæsthetics.

However, in order to model neuronal activity, deeper insight is needed i.e. considering also information about amplitude dynamics. The multidimensional Fokker-Planck equation or, equivalently, the associated Langevin equation can be used to model dissipative dynamical systems under the influence of noise [2]. Therefore, we analyze δ- and θ-waves based on specific characteristics of the one- and two-dimensional Kramers-Moyal coefficients to obtain the deterministic and stochastic parts of their interaction.

Finally, we apply stability analysis to the deterministic parts [3]. The fixed points and their dynamical exponents clearly reveal different interactions between the δ- and θ-waves in deep and light anæsthesia. We expect that the approach can be generalized to all time scales of the brain waves.

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Fingerprints of Random Flows

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We consider the patterns formed by small rod-like objects advected by a random flow in two dimensions. A simple theorem indicates that their direction field is non-singular. However, we show that singular behaviour can emerge in the long time limit. First, 'flip lines' emerge where the rods abruptly change direction by π. Later, these flip lines become so narrow that they disappear, but their ends remain as point singularities. These point singularities are of the same type as those seen in fingerprints.

The Weibull - Log Weibull Transition in Interval-times of Earthquakes with Short Time Correlated Earthquakes Removed

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Analyzing seismic catalogs, for example the Japan Meteorological Agency Earthquake Catalog (JMA) and the Southern California Earthquake Catalog, and a two-dimensional spring-block model, we have revealed that the probability distributions of time intervals between successive earthquakes, inter-occurrence times, exhibit the transition from the Log Weibull regime to the Weibull one when the threshold of magnitude is increased. Based on the renewal theoretical analysis, we can conclude that the occurrence of earthquakes without correlated earthquakes is stationary because the mean and the second moment of the Weibull and the Log Weibull distribution are finite.

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Triangle map and its ergodic properties

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The *triangle map* on the torus is a non-hyperbolic system featuring properties common to mostly chaotic systems such as diffusion, ergodicity and mixing. These non-obvious properties make this map interesting also in the quantum picture. Here we present some new views on the ergodicity and on the mixing in this system, which are yet not proved. The properties of the triangle system are studied by symbolically encoding the time evolution using two different schemes called *polygonal and binary description.* At a given time t, the points of the same code form disjoint partitions on the phase space. The number of these partitions, called *topological complexity*, grows with increasing time as $O(t^3)$. In general the properties of partitions scale in some way with t, which were closely examined. We also calculated the transition probabilities between partitions, referred to as the *Markov matrix of the map*, and study its spectral gap. The gap is shrinking with increasing time as $O(t^p)$, $p > 0$, which is partially explained by the presented simple random model for the Markov matrix.

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The effect of low frequency oscillations on cardiorespiratory synchronization

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The cardiac and respiratory systems behave in an oscillatory manner, and can be regarded as two weakly coupled, self-sustained oscillators. They have been shown to interact, to the extent that they can undergo episodes of synchronization (Schäfer *et al.* 1998, Lotrič & Stefanovska 2000, Pikovsky *et al.* 2001). Using non-linear dynamics, we can obtain further insight into different physiological states, such as stages of anaesthesia (Musizza *et al.* 2007). Here we look at how this interaction is affected by a perturbation in the form of physical exercise, by comparing the synchronization episodes of subjects when lying in a resting state with those during exercise on an exercise bike. We observe the changes to the oscillators that exercise causes by wavelet analysis, in particular an increase in modulation of the respiratory frequency, and how synchronization between the two oscillators is affected. It can be seen that the cardiorespiratory system undergoes transitions by switching between different close ratios of synchronization. Using a model of phase-coupled oscillators, we show how these transitions are mainly due to modulation by low frequency components of the oscillations (Kenwright *et al.* 2008).

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ON DYNAMICS IN SOME DISCRETE QUADRATIC SYSTEMS IN THE PLANE USING THE ALGEBRAIC APPROACH

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We consider the dynamics in some special cases of quadratic homogeneous discrete dynamical systems of the form

$$\begin{matrix} x_{k+1} = a_1x_k^2 + 2b_1x_ky_k + c_1y_k^2 \\ y_{k+1} = a_2x_k^2 + 2b_2x_ky_k + c_2y_k^2 \end{matrix} \ ; \ a_i, b_i, c_i \in \mathbb{R} \text{ for } i = 1, 2. \tag{2}$$

It is well known that homogeneous quadratic maps (2) are in one to one correspondence with two-dimensional commutative (nonassociative) algebras (Markus 1960):

$*$	$\vec{e}_1$	$\vec{e}_2$
$\vec{e}_1$	$a_1\vec{e}_1 + a_2\vec{e}_2$	$b_1\vec{e}_1 + b_2\vec{e}_2$
$\vec{e}_2$	$b_1\vec{e}_1 + b_2\vec{e}_2$	$c_1\vec{e}_1 + c_2\vec{e}_2$

; $a_i, b_i, c_i \in \mathbb{R}$ for $i = 1, 2$.

Algebraic concepts (such as structure of algebra and existence of special elements like idempotents and nilpotents) help us to study the dynamics of the corresponding discrete homogeneous quadratic maps. It is well-known that such systems can exhibit chaotic behavior (Kutnjak 2007). The simplest example is the complex-squaring map, which exhibits chaotic behavior on the unit circle which is the boundary, ∂B, of the set of all points with bounded forward orbits. We want to consider case-by-case (up to algebraic isomorphism) all commutative algebras in the plane in order to prove or disprove the existence of chaotic behavior in the corresponding discrete homogeneous quadratic system. Therefore, in this report we consider the sets ∂B (i.e. the generalized Julia sets) in some discrete homogeneous quadratic systems. We will describe the action of some structural algebraic properties and the existence of some special algebraic elements on the dynamics of the corresponding discrete homogeneous quadratic system. Some original results of the authors can be found in (Kutnjak and Mencinger 2008, Mencinger and Kutnjak).

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Exponential Stability Estimates for Trojan Asteroids - Nekhoroshev Theorem meets Celestial Mechanics

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The concept of exponential stability in nonlinear dynamical systems can be traced back to 1955 (Moser) and Littlewood (1959). It was overshadowed for decades by the KAM theorem (Kolmogorov 1954, Arnold 1963, Moser 1962), which asserts stability for all times of those orbits, with initial conditions belonging to a Cantor set of tori of non-zero measure. In 1977 Nekhoroshev revived the research on it and analyzed in great detail the exponential stability times for general Hamiltonian systems near to integrable. Exponential (sometimes called practical) stability is of much greater interest from the physical point of view, as it can be applied to *all* orbits in open domains of the phase space, whether they lie on an invariant torus or not. The corresponding theorem proven by Nekhoroshev (1977) defines stability regions for a finite time T in both, regular and chaotic domains of the phase space. If the life-time of the physical system is shorter than the stability time derived from the Nekhoroshev estimates of the region, one can definitely say that orbits belonging to this region are stable from the practical point of view. This is the reason, why the Nekhoroshev theorem has to be considered at least as important as the KAM theorem as regards its relevance to the understanding of nonlinear dynamics. I will introduce the Nekhoroshev theory in short and show one typical application in Celestial Mechanics, where the mathematical theorem can reveal physical insights into the system, namely the motion near the 1:1 resonance of the elliptic restricted three body problem (Efthymiopoulos 2005, Lhotka et al. 2008).

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HOW WELL CAN ONE RESOLVE THE STATE SPACE OF A CHAOTIC FLOW?

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All physical systems are affected by some noise that limits the resolution that can be attained in partitioning their state space. For chaotic, locally hyperbolic flows, this resolution depends on the interplay of the local stretching/contraction and the smearing due to noise. Our goal is to determine the finest possible partition of the state space for a given hyperbolic dynamical system and a given weak additive white noise of specified strength. We test these ideas on two models: the "skew Ulam" map and the Lozi attractor. We partition the state space by computing the local eigenfunctions of the Fokker-Planck evolution operator in the neighborhood of each periodic point, and use their widths to attain an optimal resolution of the state space. In both models the finest attainable partition for a given noise covers the state space by a finite tiling, and the Fokker-Planck evolution operator is represented by a finite Markov graph.

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Transport properties of waves and particles in periodic quasi-1D waveguides with mixed phase space

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We compute wave and ray transport quantities of a periodic quasi-one dimensional wave guide whose ray (or particle) dynamics undergoes the generic (KAM structure) transition from regular to global. We calculate the spatial diffusion $\sigma_n^2(t)$ as a function of the time t, where n is the n^{th} cell of the periodic waveguide, as well as the evolution of the position distribution $\rho(n)$ and momentum distribution $\rho(p_x)$. We find that $\sigma_n^2(t) \sim t$ for global chaos and $\sigma_n^2(t) \sim t^2$ for mixed chaotic dynamics with *unidirectional* direction. For mixed bi-directional motion $\sigma_n^2(t) \sim t$ after a transient time which depends on the degree of chaoticity. We also solve the Helmholtz (Schroedinger) equation to obtain the energy band structure for infinitely long periodic waveguide and the conductance for finite waveguides for the three cases, global chaos, unidirectional mixed chaos and bidirectional mixed chaos. We analyze the wave t ransport properties in terms of the underlying classical dynamics.

ON ALGEBRAIC APPROACH IN HOMOGENEOUS QUADRATIC SYSTEMS

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There is a one-to-one correspondence between homogeneous quadratic systems and nonassociative commutative finite dimensional real algebras (Markus, 1960). It is the right hand side of the system which allows us to introduce the algebraic concept into the dynamical system. The corresponding algebra multiplication $*$ is uniquely defined by $x * y = (Q(x+y) - Q(x) - Q(y))/2$. Therefore, this kind of algebraic approach is applicable for the continuous systems, $x' = Q(x)$, as well as for the discrete systems $x_{k+1} = Q(x_k)$; $x \in \mathbb{R}^n$. In this report we discus some (dis)similarities between the continuous and discrete case. The origin is always a total degenerate critical point in the continuous case (c.f. Mencinger 2003) on one hand, and is (trivial) stable in the discrete case on the other hand. There is no chaotic behaviour in $\mathbb{R}^2$ in the continuous case on one hand. On the other hand, it is well known that there is a chaotic behaviour in some discrete cases. We will also consider some (dis)similarities concerning the above described algebraic approach. In particular we will consider the influence of the algebraic structure (for example the existence of subalgebra or an ideal) to the corresponding continuous/discrete dynamical system. The meaning of algebra isomorphism is equal in both cases and it represents the basis for the linear equivalence classification of homogeneous quadratic systems. Next, we consider how the existence of some special algebraic elements (i.e. nilpotents of rank 2 and idempotents) reflects in the dynamics of the corresponding continuous/discrete system. Finally, we consider the existence of derivations and automorphisms in the corresponding algebra in order to obtain some special orbits in the system. By applying the algebraic approach to homogeneous quadratic systems the authors already solved some interesting problems in continuous systems (c.f. Mencinger, 2003, Mencinger and Zalar, 2005, Mencinger, 2006) as well as in discrete systems (c.f. Kutnjak, 2007, Kutnjak and Mencinger, 2008). Moreover, we are sure that the interplay between algebras and dynamical systems can create some new opportunities in both areas.

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Emergence of criticality in surface corrugated waveguides

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We investigate the statistical properties of eigenvalues and eigenvectors of surface corrugated waveguides depending on the degree of complexity of their boundaries. We focus on waveguide geometries whose phase space is ergodic in the classical limit. It is shown that the transition chaos-disorder,[1] driven by increasing the complexity of boundaries, is characterized by a transition in the statistical properties that goes from gaussian to critical. We model and explain this transition by the use of the Wigner-Lorenzian Random Matrix ensemble.[2] We also define a class of quantized chaotic deterministic systems that may show the emergence of criticality. This result is expected to be related to the new intermediate type of quantum chaos introduced recently in Ref. [3].

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Nonchaotic Stagnant Motion in a Marginal Quasiperiodic Gradient System

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We present a one-dimensional dynamical system with a marginal quasiperiodic gradient, as a mathematical extension of the nonunifrom oscillator [1],

$$\dot{x} = 1 - \frac{1}{2}\cos(2\pi x) - \frac{1}{2}\cos(2\pi k x),$$

which could be implemented in a multi-junction asynmmetric SQUID modeled after the 3JJ SQUID ratchet proposed by Zapata et al. [2]. The system exhibits a nonchaotic stagnant motion, which is reminiscent of intermittent chaos. In fact, the density function of residence times near stagnation points obeys an inverse-power law due to a similar mechanism to type-I intermittency. However, contrary to the intermittent chaos, the alternation between long stagnant phases and rapid moving phases occurs not randomly but in a quasiperiodic manner. Particularly in the case of gradient with the golden ratio, the renewal of the largest residence time occurs on the positions corresponding to the Fibonacci sequence. Finally, the asymptotic long-time behavior in the form of nested logarithm is theoretically derived. In comparison with the Pomeau-Manneville intermittency, a significant difference in the relaxation property of the long-time average of dynamical variable is elucidated.

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Turbulence in Diffusion Replicator Equation

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Dynamical behaviors in the diffusion replicator equation of three species are numerically studied. We study the following replicator dynamics with diffusion,

$$\begin{cases} \dfrac{\partial X_i}{\partial t} = X_i(\sum_{j=1}^{3} g_{ij} X_j - \sum_{j=1}^{3}\sum_{k=1}^{3} g_{jk} X_j X_k) + D\dfrac{\partial^2}{\partial r^2} X_i \\ \sum_{i=1}^{3} X_i(r,t) = 1 \ \ and \ \ 0 \le X_1(r,t), X_2(r,t), X_3(r,t) \le 1, \ \ [0 \le r \le L] \end{cases} \tag{3}$$

where $X_i = X_i(r,t)$ is the frequency of i species (or player) $(i = 1,2,3)$, $G = \{g_{ij}\}$ the interaction matrix, r $\in [0, L]$ the one dim. space with periodic boundary, L the system size, and D the diffusion coefficient.Our motivation is the followings; when two or more heteroclinic cycles interact, what kind of complex behaviors come out?

Firstly, the bifurcation diagram for a certain parameter setting is drawn. Then it is shown that the turbulence appears with the supercritical Hopf bifurcation of a stationary uniform solution and it disappears under a subcritical-type bifurcation. Secondly, the statistical property of the turbulence near the supercritical Hopf onset point is analyzed precisely. Further, the correlation lengths and correlation times obey some characteristic scaling laws.

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Approach towards equilibrium in systems with long-range interactions

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Considering dynamics of many particle interacting systems, the teaching of statistical physics postulates that on the microscopic level the disorder dominates and phase-space mixing produces the most likely homogeneous final state.

In a system where particles interact over long ranges, such as in a model of a free-electron laser in Sicrotrone Trieste, our numerical investigations have provided evindence that particles tend to linger in long-lived coherent states and that the transition to the final state takes longer than expected.

How does an initial formation of particles form a coherent state and gradually migrates towards an asymptotic equilibrium? What are the mechanisms of slowing down the phase-space mixing?

We will present results of our ongoing investigation of these problems using methods of dynamical systems. The specific question we will address is how do equilibrium structures and their manifolds mold the path of density evolution and how does the dramatic story of disorder winning over structure unwind in a system, representing many particles interacting with an electromagnetic wave, the Bonifacio model of free-electon lasers.

Ergodicity and Complexity in Infinite Measure Dynamical Systems

— Scaling Laws and the Strong Intermittency of the Log-Weibull Map —

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Intemittent behaviors have been studied by use of the infinite ergodic theory [Shinkai (2006), Akimoto]. The most striking point is that inevitable statistical properties appear in the non-stationary chaos, such that the distribution of normalized Lyapunov exponents convergences to the Mittag-Leffler distribution [Aaronson, Shinkai(2007)]. In the same way, the distribution of the Lempel-Ziv complexity in infinite ergodic systems does, and the scaling laws of the Lempel-Ziv complexity are theoretically evaluated [Shinkai(2006)].

Here we study a class of one-dimensional maps with an infinite measure T called "*Log-Weibull Map with an Infinite Meaure*", which is an extended model of Thaler's example [Thaler]. The map T is given by,

$$T(x) = \begin{cases} x + \frac{1}{2}f(2x) & \text{for} \quad x \in I_0 = [0,\, 0.5), \\ x - \frac{1}{2}f(2-2x) & \text{for} \quad x \in I_1 = [0.5,\, 1], \end{cases}$$

where the function f is defined as follows:

$$f(t) = t^{1+\beta}\exp(1 - t^{-\beta}), \quad t \in [0,\, 1], \quad 0 < \beta < 1.$$

We prove that orbits generated by the map T reveal the strongest non-stationarity such as f^{-2} spectral fluctuations and the residence time distribution $P(m)$ in intervals I_0 and I_1 is the log-Weibull one,

$$P(m) \sim m^{-1}(\log m)^{-1-\frac{1}{\beta}}.$$

As the Darling-Kac-Aaronson theorem in infinite ergodic theory says [Darling-Kac, Aaronson], the normalized partial sum of L^1_+ function $\frac{S_N}{a(N)} = \frac{1}{a(N)}\sum_{i=0}^{N-1} g \circ T_s$ $(g \in L^1_+)$ is the random variable which obeys the Mittag-Leffler distribution as $N \to \infty$, where $a(N)$ is regularly varying at ∞ with index $\alpha = 0$ theoretically. Moreover, we show numerical and/or phenomenal aspects of the strongest non-stationarity such as $\alpha \to 0$ $(N \to \infty)$

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On the theory of cavities with point-like perturbations

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The theoretical interpretation of measurements of "wavefunctions" and spectra in electromagnetic cavities excited by antennas is considered. Assuming that the characteristic wavelength of the field inside the cavity is much larger than the radius of the antenna, we describe antennas as "point-like perturbations". This approach strongly simplifies the problem reducing the whole information on the antenna to four effective constants. In the framework of this approach we overcame the divergency of series of the phenomenological scattering theory and justify assumptions lying at the heart of "wavefunction measurements". This selfconsistent approach allowed us to go beyond the one-pole approximation, in particular, to treat the experiments with degenerate states. The central idea of the approach is to introduce "renormalized" Green function, which contains the information on boundary reflections and has no singularity inside the cavity.

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Expanded boundary integral method and chaotic time-reversal doublets in quantum billiards

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We present the expanded boundary integral method for solving the planar Helmholtz problem, which combines the ideas of the boundary integral method and the scaling method and is applicable to arbitrary shapes. We apply the method to a chaotic billiard with unidirectional transport, where we demonstrate existence of doublets of chaotic eigenstates, which are quasi-degenerate due to time-reversal symmetry, and a very particular level spacing distribution that attains a chaotic Shnirelman peak at short energy ranges and exhibits GUE-like statistics for large energy ranges. We show that, as a consequence of such particular level statistics or algebraic tunneling between disjoint chaotic components connected by time-reversal operation, the system exhibits quantum current reversals.

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