

# Abstracts of Posters

# CYCLE EXPANSIONS OF A TWO-DIMENSIONAL PIECEWISE LINEAR MAP

Akira Akaishi and Akira Shudo

*Department of Physics, Tokyo Metropolitan University, Minami-Ohsawa, Hachioji, Tokyo 192-0397, Japan*

We investigate algebraic correlation of a certain class of two-dimensional piecewise linear map. For this map, there is a rigorous proof showing that there exists a series of parameter values at which polygon-shaped regular components and chaotic components coexist in phase space and these boundaries are strictly sharp (no resonance islands). We here present a systematic method to enumerate all the unstable periodic orbits in chaotic components with their linear stability and apply the cycle expansion method to derive the power-law behavior which manifests itself in the survival probability.

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## On the ergodic measure of the non-equilibrium non-stationary state: Generalized arc-sine law and stable law

Takuma Akimoto

*Department of Applied Physics, Advanced School of Science and Engineering, Waseda University, Okubo 3-4-1, Shinjuku-ku, Tokyo 169-8555, Japan.*

Limit theorems for the time average of non- $L^1(m)$  function in an infinite measure dynamical system are studied. We present the generalized arc-sine law and stable law in the skew modified Bernoulli map modeling the on-off intermittent phenomena.

Furthermore, we study the ergodic measure characterizing the non-equilibrium non-stationary state on the basis of the macroscopic observable, which results from the time average of the microscopic observable  $f$  in the dynamical system. Finally, we show that the generalized arc-sine distribution can be one of the ergodic measures of the non-equilibrium non-stationary state.

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# Directionality of coupling and synchronization between coupled oscillators

A. Bahraminasab, A. Stefanovska, P. V. E. McClintock

*Department of Physics, Lancaster University, Lancaster, LA1 4YB, UK*

The dynamics of many natural and man-made systems, and correspondingly the signals derived from them, is highly complex. This is especially true of the cardiovascular system and the brain. During the recent years much effort has been devoted to quantitative characterisation of the dynamical properties of complex systems by applying different time series analysis techniques. Complex dynamics has often been considered to result from oscillatory processes. Characterisation of the directionality of the couplings between different components of the system [1] and mutual synchronization [2] assume particular importance.

In this work, we present a recently introduced directionality index [3] for time series. It is based on conditional mutual information of data generated from comparison of neighboring values. We discuss the efficiency of the method and show that it can distinguish between different kinds of coupling, i.e. between unidirectional and bidirectional coupling, as well as reveal and quantify any asymmetry in bidirectional coupling. The fact that there is no need for preprocessing or fine-tuning of parameters makes the method very simple, computationally fast and robust.

Further, we introduce a new [4], simple and powerful method of detecting synchronization in noisy bivariate data. It is based on the detection of phase restriction, revealed as plateaus in plots of synchronization time as a function of a threshold defining the domain of acceptance for the phase difference. Unlike earlier methods, the criteria for fixing the optimal threshold and windows of observation arise naturally, facilitating reliable detection of synchronous epochs.

The two techniques will be illustrated using both numerical data and real data obtained from the cardiovascular system.

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## NERVE PULSE PROPAGATION IN A CHAIN OF FHN NONLINEAR OSCILLATORS

E. Christodoulidi\*, S. Anastassiou\*, J. P. van der Weele\* and T. Bountis\*

\*Center for Research and Applications of Nonlinear Systems (CRANS), Department of Mathematics,  
University of Patras, GR-26500, Patras, Greece.

A particularly useful and instructive model for the study of nerve pulse propagation is described by the well - known FitzHugh Nagumo (FHN) partial differential equations. In the absence of diffusion, the FHN system represents a single point - like neuron and is expressed in terms of two Ordinary Differential Equations (ODEs) for the membrane electric potential and the recovery (ion) current. In this work, we connect  $N$  such FHN oscillators in a one - directional way, using the same coupling constant  $\alpha$ . We then apply to the first ODE a periodic square wave of period  $T$ , amplitude  $A$  and duration  $\Delta T$ , sufficient to excite the first neuronal oscillator. We then investigate ranges of parameter values for which the excited action potential wave train is transmitted to the subsequent FHN oscillators of the chain with the same period  $T$ . We thus discover conditions under which the transmitted wave has a period approximately equal to  $2T$  or  $3T, \dots$ , or fails to be transmitted far enough. We then add diffusion and also solve the FHN partial differential equations numerically to explore the effect of external forcing on the propagation of spatially extended pulses, which resemble more closely the action potential waves one encounters in actual nerve pulse propagation and cardiac muscle fiber contractions.

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## USE OF NORMALIZED RADIAL BASIS FUNCTION IN HYDROLOGY

Anton Čotar

FGG and FMF, University of Ljubljana, Slovenia

We used normalized radial basis function (NRBF) for reconstruction of river Reka runoff. We have measured data of Reka monthly runoff for period 1952-2006. By using precipitation and temperature data from Trieste (Italy) for period 1851-2006 we calculated numbers, that we interpret as Reka runoff for period 1851-1951. For prediction process we used multidimensional normal distribution, whose standard deviation was optimized on data from period 1952-1990. That was the learning process. The verification was done on period 1991-2006. For coefficient of quality of prediction we get 0,85. With optimized multidimensional normal distribution we then calculated a 1851-1951 vector, that represents Reka runoff for the same period. The problems with minimal and maximal runoff are still present, but this was expected, because the method is conservative. We get to the conclusions that this method is very useful for practical applications. It is more physical than often used linear regression and machine learning methods, because it is based on principle of measurement errors and maximal entropy.

# VARIABLE-RANGE PROJECTION MODEL FOR TURBULENCE-DRIVEN COLLISIONS

**Kristian Gustavsson**

*Department of Physics, Göteborg University, 41296 Gothenburg, Sweden*

We discuss the probability distribution of relative speed  $\Delta V$  of inertial particles suspended in a highly turbulent gas when the Stokes numbers, a dimensionless measure of their inertia, is large. We identify a mechanism giving rise to the distribution  $P(\Delta V) \sim \exp(-C|\Delta V|^{4/3})$  (for some constant  $C$ ). Our conclusions are supported by numerical simulations and the analytical solution of a model equation of motion. The results determine the rate of collisions between suspended particles. They are relevant to the hypothesised mechanism for formation of planets by aggregation of dust particles in circumstellar nebula.

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# Local first integrals of a cubic system of differential equations

Zhibek Kadyrsizova

*al-Farabi Kazakh National University,  
Faculty of Mechanics and Mathematics, Almaty, Kazakhstan*

We consider a polynomial system of differential equations of the form

$$\dot{x} = x + p(x, y), \quad \dot{y} = -3y + q(x, y), \quad (1)$$

where  $p(x, y)$  and  $q(x, y)$  are homogeneous polynomials of degree three. In this paper we look for the first integrals of the system using as the main tool the Darboux method. In (Kadyrsizova and Romanovski) the linearizability problem for system (1) has been studied. In many cases considered there the construction of linearizing transformations require the knowledge of first integrals. In (Hu et al) it has been proved that such integrals exist. In the present paper we find the explicit expressions for the integrals. It allows to obtain the explicit formulas for the corresponding linearizing transformations of (Kadyrsizova and Romanovski). To perform the study we have developed some algorithms (and have implemented them in MAPLE) for finding invariant curves and their cofactors. Using the invariant curves we have constructed the local first integrals of system (1).

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# Critical Flow and Pattern Formation of Granular Matter on a Conveyor Belt

Georgios Kanellopoulos and Ko van der Weele

*Mathematics Department,  
University of Patras,  
26500 Patras, Greece*

We study the flow of granular material on a conveyor belt consisting of  $K$  connected, vertically vibrated compartments. A steady inflow is applied to the top compartment and our goal is to describe the conditions that ensure a continuous flow all the way down to the  $K$ th compartment. In contrast to normal fluids, flowing granular matter has a tendency to form clusters (due to the inelasticity of the particle collisions [Goldhirsch and Zanetti, 1993]); when this happens the flow stops and the outflow from the  $K$ th compartment vanishes.

Given the dimensions of the conveyor belt and the vibration strength, we determine the critical value of the inflow beyond which cluster formation is inevitable. Fortunately, the clusters are announced in advance (already below the critical value of the inflow) by the appearance of a wavy density profile along the  $K$  compartments. The critical flow and the associated wavy profile are explained quantitatively in terms of a dynamical flux model [Eggers, 1999; Van der Weele *et al.*, 2001]. This same model enables us to formulate a method to greatly increase the critical value of the inflow, improving the capacity of the conveyor belt by a factor two or even more.

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# Paced respiration - furthering the understanding of cardio-respiratory synchronization and modulation during anaesthesia

D. A. Kenwright and A. Stefanovska

*Department of Physics, Lancaster University, UK*

The cardio-respiratory system is an example of two coupled nonlinear oscillators that occur in nature, and provide the possibility to learn about characteristic phenomena. Of these, two are known: synchronization and modulation.

It is possible to perturb this system and observe the changes; for instance exercise increases the frequencies of both oscillators and effectively destroys synchronization and reduces modulation (Kenwright *et al.* 2008). Another perturbation can be achieved with anaesthesia, whereby the variability of the oscillators is reduced and we observe an increase in synchronization and modulation, as well as transitions in the synchronization ratio depending on the stage of anaesthesia (Stefanovska *et al.* 2000). From this it has been hypothesized that synchronization analysis may provide a means of controlling the depth of anaesthesia (Musizza *et al.* 2007).

Often during anaesthesia, respiration must be assisted by some mechanical respirator. Because of this, the question arises as to how the cardiorespiratory interaction changes if one of the oscillators (in this case respiration) is fixed. To answer this question, we obtain measurements from volunteers who breathe in time with a metronome. This is carried out for 3 different frequencies, above, below and approximately equal to the natural breathing frequency, and we compare with spontaneous breathing. Here we present results of analysis obtained from wavelet analysis (Bračič & Stefanovska 1999) and recently developed algorithms for detecting synchronization and determining directionality (Bahraminasab *et al.* 2008).

This work is part of the Brain, Respiration And Cardiac Causalities In Anaesthesia (BRACCIA) project, a Europe-wide interdisciplinary project involving anaesthetists, physiologists, neuroscientists, biomedical and electrical engineers, information theorists and physicists, with the aim being to reduce the incidence of awareness during anaesthesia.

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## Complex chaos in the conditional dynamics of qubits

T. Kiss<sup>(1)</sup>, I. Jex<sup>(2)</sup>, G. Alber<sup>(3)</sup>, T. Langrová<sup>(2)</sup> and S. Vymětal<sup>(2)</sup>

(1) *Research Institute for Solid State Physics and Optics, H-1525 Budapest, P. O. Box 49, Hungary*

(2) *Department of Physics, FJFI ČVUT, Břehová 7, 115 19 Praha 1 - Staré Město, Czech Republic*

(3) *Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*

**Abstract.** The presence of complex chaos in iterative application of selective dynamics on quantum systems is a novel form of quantum chaos with true sensitivity to initial conditions. We present results for an ensemble of single qubits demonstrating how an efficient purification process can be destroyed due to the appearance of chaotic oscillations. The Julia sets of the studied process show a complicated structure with shapes strongly varying in dependence on the parameters of the dynamic. Techniques for the study of pure states are extended to include the cases of mixed states and entangled states.

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# Transmission properties of the positional disordered photonic Kronig-Penney model. Experiment and theory

G.A. Luna-Acosta, N.Makarov, F.M. Izrailev

*Instituto de Fisica, Universidad Autónoma de Puebla, Apdo. Postal J-48, CP. 72570, Puebla, México,*

U. Kuhl, and H-J. Stöckmann

*Fachbereich Physik, Philipps Universität Marburg, Renthof 5, D-35031, Germany*

We study the phenomena of localization in the transmission of a single mode through a positional disordered array formed by two alternating dielectrics of dielectric constants  $n_1$  and  $n_2 \neq n_1$ . The disorder is realized by randomly varying the width of one of the two dielectrics and keeping constant the width of the other. In the absence of disorder, this system is a photonic counterpart of the quantum Kronig-Penney model (\*). Under any amount of disorder there is complete transparency at frequencies such that  $kd_2 = m\pi$ , where  $k(\nu)$  is the wave vector at frequency  $\nu$  and  $d_2$  is the width of dielectric of constant width. In the regime of weak uncorrelated disorder we derive an analytical formula for the inverse localization length  $L_{loc}^{-1}$  and the average of the logarithm of the transmission, which is in perfect agreement with straight-forward transfer matrix calculations and with the transmission data of a micro-wave experiment.

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## ON MOVING DISCRETE LOCALIZED MODES IN NONLINEAR LATTICES

Aleksandra Maluckov<sup>1</sup>, Milutin Stepić<sup>2</sup>, Goran Gligorić<sup>2</sup>, Ljupčo Hadžievski<sup>2</sup>, and Boris Malomed<sup>3</sup>

<sup>1</sup> *Faculty of Sciences and Mathematics, P.O.B. 224, 18001 Niš, Serbia*

<sup>2</sup> *Institute for Nuclear Sciences-Vinča, P.O.B. 522, 11001 Belgrade, Serbia*

<sup>3</sup> *Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel*

Dynamics of discrete localized modes in nonlinear lattice like systems has been attracted a huge research interest in the nonlinear physics. It is worth to mention nonlinear optics, Bose-Einstein condensation phenomena, biophysics, solid state physics etc. Our investigations in this field are based on the interpretation of the localized mode dynamics in diverse nonlinear optical lattices, or lattices with different types of nonlinearity: cubic, cubic-quintic, saturable, etc. In addition we shown that the same methods are applicable to the other nonlinear problem as the Bose-Einstein condensates in deep periodic potential. The special interest has been the intriguing possibility to manipulate motion of localized intrinsic structures of high power in all these media. The concern was to interpret this possibility with respect to the correlation between the effect of nonlinearity and discreteness. Two general approaches are numerically tested: the free energy concept and mapping analysis. Only the second one has been shown appropriate in all studies.

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# Nonchaotic Stagnant Motion in a Marginal Quasiperiodic Gradient System

Takahito Mitsui

*Department of Applied Physics, Faculty of Science and Engineering, Waseda University, Tokyo 169-8555, Japan*

We present a one-dimensional dynamical system with a marginal quasiperiodic gradient, as a mathematical extension of the nonuniform oscillator [1],

$$\dot{x} = 1 - \frac{1}{2} \cos(2\pi x) - \frac{1}{2} \cos(2\pi kx),$$

which could be implemented in a multi-junction asymmetric SQUID modeled after the 3JJ SQUID ratchet proposed by Zapata et al. [2]. The system exhibits a nonchaotic stagnant motion, which is reminiscent of intermittent chaos. In fact, the density function of residence times near stagnation points obeys an inverse-power law due to a similar mechanism to type-I intermittency. However, contrary to the intermittent chaos, the alternation between long stagnant phases and rapid moving phases occurs not randomly but in a quasiperiodic manner. Particularly in the case of gradient with the golden ratio, the renewal of the largest residence time occurs on the positions corresponding to the Fibonacci sequence. Finally, the asymptotic long-time behavior in the form of nested logarithm is theoretically derived. In comparison with the Pomeau-Manneville intermittency, a significant difference in the relaxation property of the long-time average of dynamical variable is elucidated.

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## Turbulence in Diffusion Replicator Equation

Kenji Orihashi and Yoji Aizawa

*Department of Applied physics, Waseda University,  
3-4-1 Okubo, Shinjuku, Tokyo 169-8555, Japan*

Dynamical behaviors in the diffusion replicator equation of three species are numerically studied. We study the following replicator dynamics with diffusion,

$$\left\{ \begin{array}{l} \frac{\partial X_i}{\partial t} = X_i \left( \sum_{j=1}^3 g_{ij} X_j - \sum_{j=1}^3 \sum_{k=1}^3 g_{jk} X_j X_k \right) + D \frac{\partial^2}{\partial r^2} X_i \\ \sum_{i=1}^3 X_i(r, t) = 1 \text{ and } 0 \leq X_1(r, t), X_2(r, t), X_3(r, t) \leq 1, \quad [0 \leq r \leq L] \end{array} \right. \quad (2)$$

where  $X_i = X_i(r, t)$  is the frequency of  $i$  species (or player) ( $i = 1, 2, 3$ ),  $G = \{g_{ij}\}$  the interaction matrix,  $r \in [0, L]$  the one dim. space with periodic boundary,  $L$  the system size, and  $D$  the diffusion coefficient. Our motivation is the followings; when two or more heteroclinic cycles interact, what kind of complex behaviors come out?

Firstly, the bifurcation diagram for a certain parameter setting is drawn. Then it is shown that the turbulence appears with the supercritical Hopf bifurcation of a stationary uniform solution and it disappears under a subcritical-type bifurcation. Secondly, the statistical property of the turbulence near the supercritical Hopf onset point is analyzed precisely. Further, the correlation lengths and correlation times obey some characteristic scaling laws.

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## Dynamical bottlenecks to intramolecular energy flow

**R. Paškauskas, C. Chandre, T. Uzer**

*Sincrotrone Trieste, AREA Science Park, S. S. 14 KM 163.5, 34012 Basovizza TS, Italy  
Centre de Physique Théorique - CNRS, Luminy - Case 907, 13288 Marseille cedex 09, France  
Center for Nonlinear Sciences, School of Physics, Georgia Institute of Technology, Atlanta, Georgia  
30332-0430, U.S.A.*

Vibrational energy flows unevenly in molecules, repeatedly going back and forth between trapping and roaming. We identify bottlenecks between diffusive and chaotic behavior, and describe generic mechanisms of these transitions, taking the carbonyl sulphide molecule OCS as a case study. The bottlenecks are found to be lower-dimensional tori; their bifurcations and unstable manifolds govern the transition mechanisms.

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R. Paškauskas and C. Chandre and T. Uzer 2008 *Phys. Rev. Lett.* **100** (8) 083001(4)

## Two-body random spin ensemble: a new type of quantum phase transition

**Iztok Pižorn, Tomaž Prosen, and Thomas H. Seligman**

*University of Ljubljana, Faculty of Mathematics and Physics, Department of Physics, Jadranska 19, SI-1000  
Ljubljana  
Centro Internacional de Ciencias, Apartado postal 6-101, C.P.62132 Cuernavaca, Morelos, Mexico*

Random matrix models were introduced about fifty years ago by Eugene Wigner. The scope of applications has increased over the years including various fields in physics, more recently also quantum information theory where the concept of individual qubits and their interactions becomes important. In quantum information theory, however, qubits are taken to be distinguishable and it is very pertinent to formulate and investigate two-body random ensembles (TBRE) for such a case.

In the present work we study properties of such spin TBRE's by analyzing a parametrization in terms of the group parameters and the remaining parameters associated with the “entangling” part of the interaction. Using symmetry arguments we propose an adequate definition for such ensembles in a very general framework in terms of independent Gaussian distributed variables.

In order to show the relevance of the new ensemble we address the simplest topology, namely the quantum chain with nearest neighbor interactions. We focus on the ensemble averaged structure of the ground state described by the entanglement measures and spin correlations and demonstrate the existence of an unusual quantum phase transition (QPT) which is triggered by breaking of time-reversal invariance.

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Pižorn I, Prosen T, Mossmann S, Seligman T H 2008 *New J. Phys.* **10** 023020.

# Ergodic Properties for the Log-Weibull Map with a Uniform Measure

Soya Shinkai and Yoji Aizawa

Department of Applied Physics, Faculty of Science and Engineering,  
Waseda University, Tokyo 169-8555, Japan

Recently, the study of one-dimensional maps is developed for the understanding of Hamiltonian systems [Miyaguchi, Pikovskij], where both the hyperbolic region and the non-hyperbolic one coexist with a uniform Lebesgue measure. Aizawa showed that a universal statistical law is theoretically derived from the Nekoroshev theorem [Aizawa]. The law is that the probability density for the pausing time around KAM tori obeys the log-Weibull distribution asymptotically.

According to their studies, we study a class of one-dimensional maps  $T$  given by,

$$T(x) = \begin{cases} x + (1-c)f\left(\frac{x}{c}\right) & \text{for } x \in I_0 = [0, c), \\ x - c + cf^{-1}\left(\frac{x-c}{1-c}\right) & \text{for } x \in I_1 = [c, 1]. \end{cases}$$

We can prove that a uniform measure is derived from the Frobenius-Perron equation. Here we consider the case of  $c = (2\beta + 1)/(2\beta + 2)$  and the function

$$f(t) = t^{1+\beta} \exp(1 - t^{-\beta}), \quad t \in [0, 1], \quad 0 < \beta < 1.$$

Then the map is called “Log-Weibull Map with a Uniform Measure”. (In the case  $f(t) = t^{\frac{\beta}{\beta-1}}$  ( $\beta > 1$ ), the map is consistent with Miyaguchi’s one.) In numerical calculations, the inverse function  $f^{-1}$  needs to be written explicitly by use of the “Lambert  $W$  function” which is defined as  $z = W(z)e^{W(z)}$  [Corless]. We derived the inverse function explicitly

$$f^{-1}(t) = \left\{ \frac{1+\beta}{\beta} W\left(\frac{\beta}{1+\beta} e^{\frac{\beta}{1+\beta}} t^{-\frac{\beta}{1+\beta}}\right) \right\}^{-\frac{1}{\beta}}.$$

Unlike the log-Weibull map with an infinite measure, in this map the residence time distribution  $P(m)$  in the interval  $I_0$  obeys the differentiated log-Weibull distribution,

$$P(m) \sim m^{-2}(\log m)^{-1-\frac{1}{\beta}} \left\{ 1 + \frac{1+\beta}{\beta} (\log m)^{-1} \right\}.$$

The power spectral density reveals the following form:  $S(\omega) \sim \omega^{-1}(-\log \omega)^{-\frac{1}{\beta}}$  ( $\omega \ll 1$ ), and the correlation function  $C(\tau) \sim (\log \tau)^{-\frac{1}{\beta}}$  ( $\tau \gg 1$ ), which are consistent with theoretical calculations. Finally, We briefly discuss the interrelation between the log-Weibull map  $T$  and the Hamiltonian chaos [Kikuchi].

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## Effects of point-like perturbations in billiards

Timur Tudorovskiy, Ruven Höhmann, Ulrich Kuhl and Hans-Jürgen Stöckmann

*FB Physik, Philipps-Universität, Renthof 5, D-35032 Marburg, Germany*

We consider experiments with microwave cavities perturbed by point-like couplings. Using the general approach we describe several types of experiments and discuss different effects of the perturbation. In the presented theory the central place belongs to the “renormalized” Green function.

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## Complex chaos in the conditional dynamics of qubits

T. Kiss<sup>(1)</sup>, I. Jex<sup>(2)</sup>, G. Alber<sup>(3)</sup>, T. Langrová<sup>(2)</sup> and S. Vymětal<sup>(2)</sup>

(1) *Research Institute for Solid State Physics and Optics, H-1525 Budapest, P. O. Box 49, Hungary*

(2) *Department of Physics, FJFI ČVUT, Břehová 7, 115 19 Praha 1 - Staré Město, Czech Republic*

(3) *Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany*

**Abstract.** The presence of complex chaos in iterative application of selective dynamics on quantum systems is a novel form of quantum chaos with true sensitivity to initial conditions. We present results for an ensemble of single qubits demonstrating how an efficient purification process can be destroyed due to the appearance of chaotic oscillations. The Julia sets of the studied process show a complicated structure with shapes strongly varying in dependence on the parameters of the dynamic. Techniques for the study of pure states are extended to include the cases of mixed states and entangled states.

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# An Analytical Tractable Model for Dynamic Action Potential Encoding in Spatially Extended Neurons

W. Wei<sup>1,2</sup>, F. Wolf<sup>1,2,3</sup>

<sup>1</sup> *MPI for Dynamics and Self-Organization,*

<sup>2</sup> *Bernstein Center for Computational Neuroscience,*

and <sup>3</sup> *Faculty of Physics, University of Goettingen, Goettingen, Germany*

In the cerebral cortex, the results of all neuronal operations performed at the single cell level are coded into sequences of action potentials (APs). In the living brain, cortical neurons are subject to an immense synaptic bombardment, resulting in large fluctuations of their membrane potential and in temporally irregular AP firing. Recently, the AP encoding under conditions of such synaptic bombardment has received much attention and has been analyzed extensively in single compartment neuronal models. Real neurons, however, are spatially extended systems and physiological studies indicate that the site of action potential initiation of cortical neurons is located in a relatively small neuronal process, the proximal part of the axon. The impact of this geometry on AP wave form and AP encoding is a matter of ongoing controversy. To elucidate the impact of axonal AP initiation, we here take the axon as a semi-infinite cable and calculate the transfer of voltage fluctuation in the axon using the Green's function method. In the framework of Gaussian neuron model we obtain the spike-triggered voltage and variance at the soma when a spike is triggered at axon. We find that the spike-triggered variance is very small compared with the experimentally observed variability of thresholds at the soma. We also study the linear response function for the dynamical firing rate when action potentials are elicited in the axon and a small sinusoidal current is injected at soma.

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## Topology of chaotic scattering in four effective dimensions.

Christoff Jung and Karel Zapfe

*Instituto de Ciencias Fisicas*

*UNAM, Av. Universidad s/n, Col. Chamilpa, Cuernavaca, Morelos, 62210, Mexico.*

We will present a generalization to situations with four effective degrees of freedom of the Smalle Horseshoe construction. The goal of the approach will be finding relevant topological characteristics of the chaotic saddle and the invariant manifolds on data that can be experimentally measured. The method is basically the one used for the two dimensional case: We will study the topology of the singularity subsets of the scattering functions and we will try to find a way to correlate these to the rainbow singularities of effective cross section data.