Quantum Measurement without Schrödinger cat states

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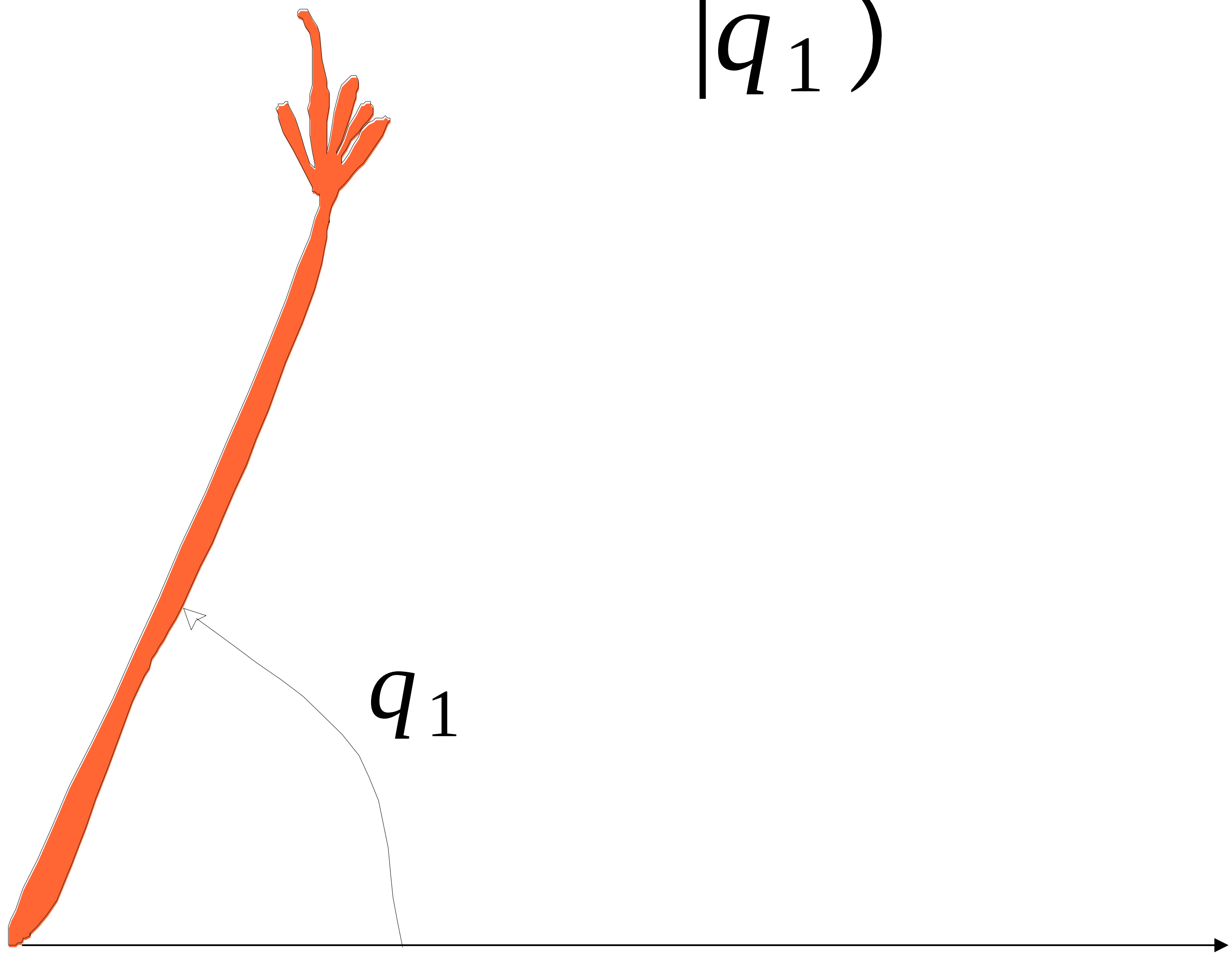
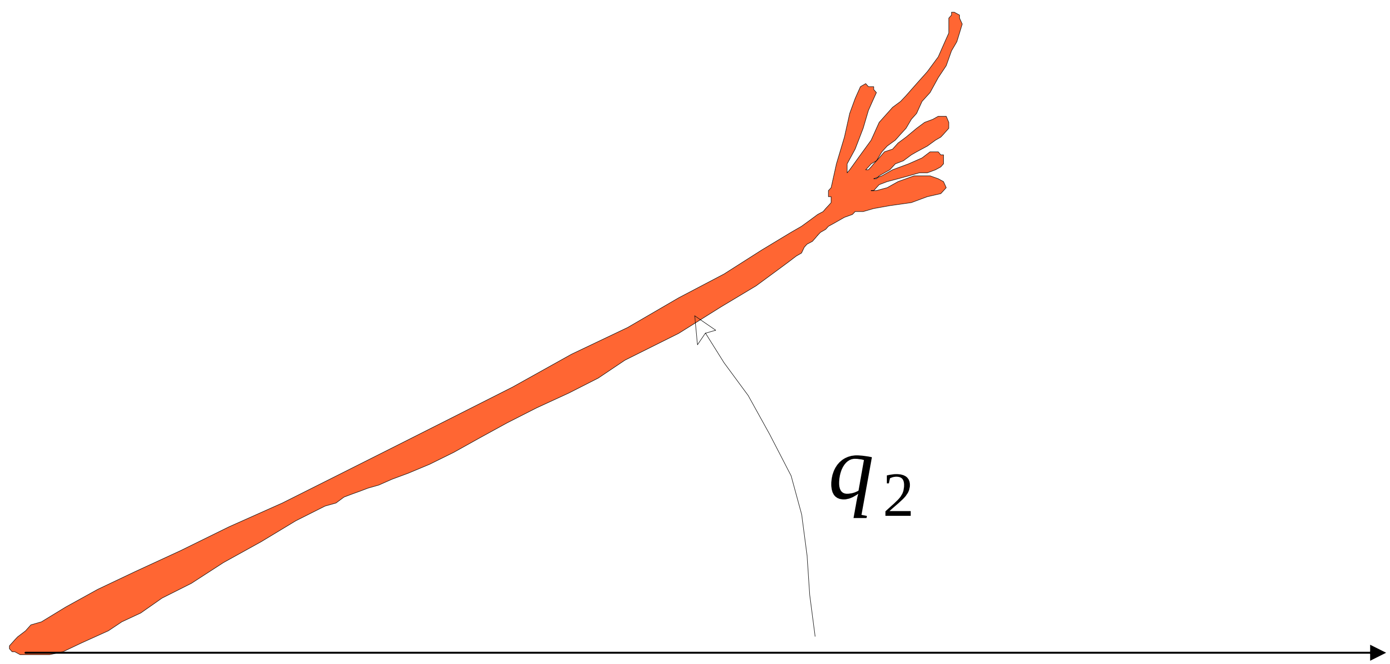
intro

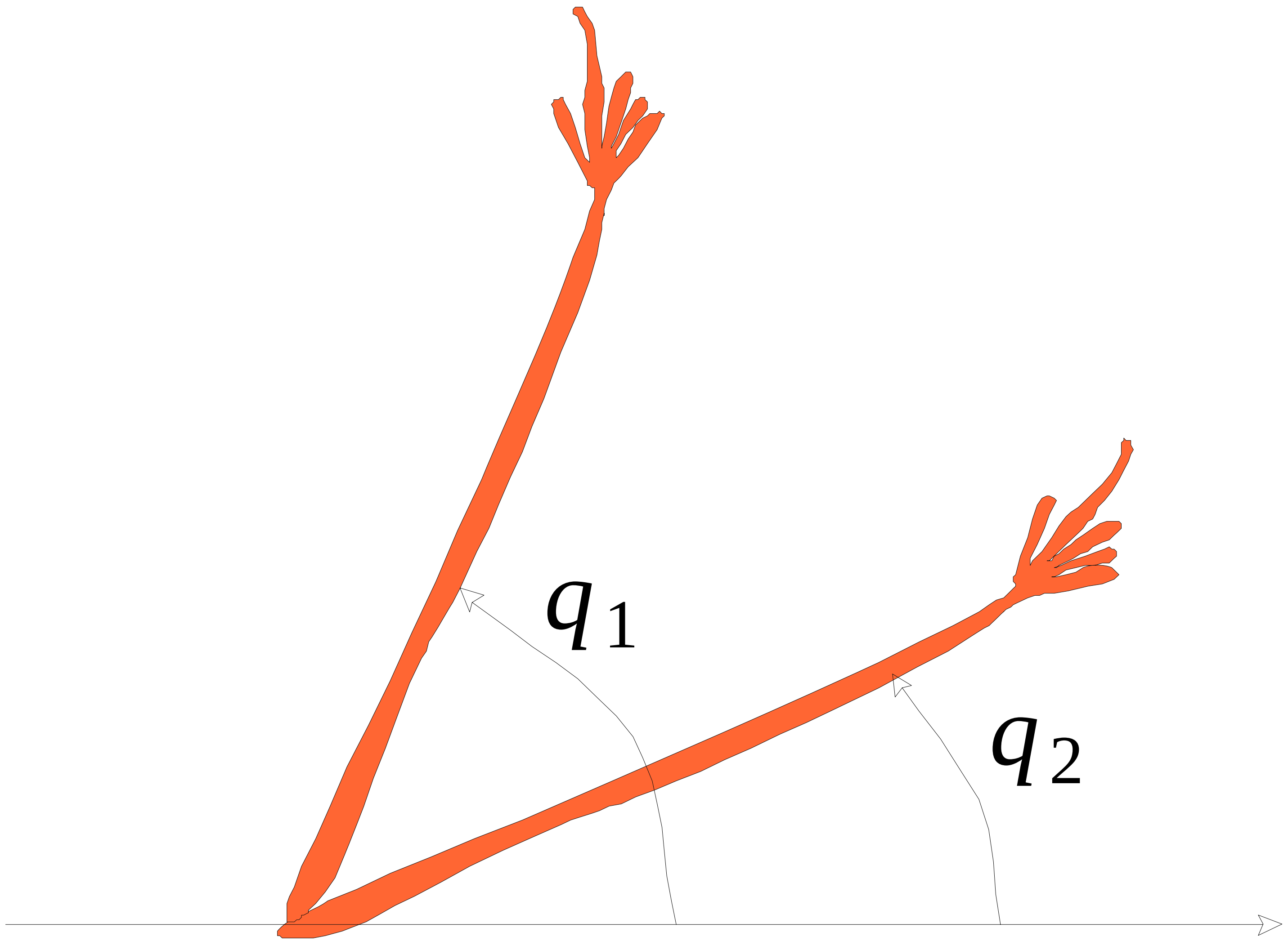
$$|q_1\rangle$$



$|q_2)$



why never interferences from $\varphi_1 |q_1\rangle + \varphi_2 |q_2\rangle$?



“axiom”

if observable $\hat{\xi} = \sum_n \xi_n |n\rangle\langle n|$ measured

for pure state $|\varphi\rangle = \sum_n \varphi_n |n\rangle$

single run yields some unpredictable ξ_n

many runs: ξ_n with probability $|\varphi_n|^2$

ensemble left in mixed state $\sum_n |\varphi_n|^2 \, |n\rangle\langle n|$

old lore: collapse of pure state to mixture

incompatible with unitary time evolution

state-of-the-art lore

“axiom” degraded to solution of
Schrödinger eqn for object and apparatus

exactly solvable models reveal:

different ξ_n entangled with macroscopically distinct pointer displacements

decoherence of different pointer displacements simplest model

object O + pointer P + bath B

observable $\hat{\xi}$

single freedom, macroscopic

$[\hat{q},\hat{p}] = i\hbar$

many freedoms

$$H = \cancel{H_O} + \cancel{H_P} + \cancel{H_B} + H_{OP} + H_{PB}$$

entanglement

decoherence

$$\rho_{OPB}(0) = |\varphi\rangle\langle\varphi| \otimes \rho_{PB}(0)$$

exactly solvable if harmonic oscillators for P and B
and suitable choices for the interactions

for now, forget exact solution, assume entanglement and decoherence fastest

initially thermal pointer

if pointer harmonic oscillator, initially thermal,

rms pointer displacement $\Delta q = \sqrt{kT/m\omega} \approx 10^{-10} m$

de Broglie wavelength $\lambda = \hbar/\sqrt{mkT} \approx 10^{-22} m$

for $m = 1g,\ \omega = 1\,\mathrm{sec}^{-1},\ T = 300K$

that's a macroscopic pointer!

entanglement

object pointer interaction $H_{OP} = \varepsilon \hat{\xi} \hat{p}$

momentum generates pointer displacement

proportional to eigenvalue of measured observable

coupling so strong that different eigenvalues of $\hat{\xi}$
entail macroscopically distinct pointer displmts

Schrödinger cat state

would be produced by H_{OP} alone, different ξ_n
entangled with macroscop'ly distinct pointer displmts

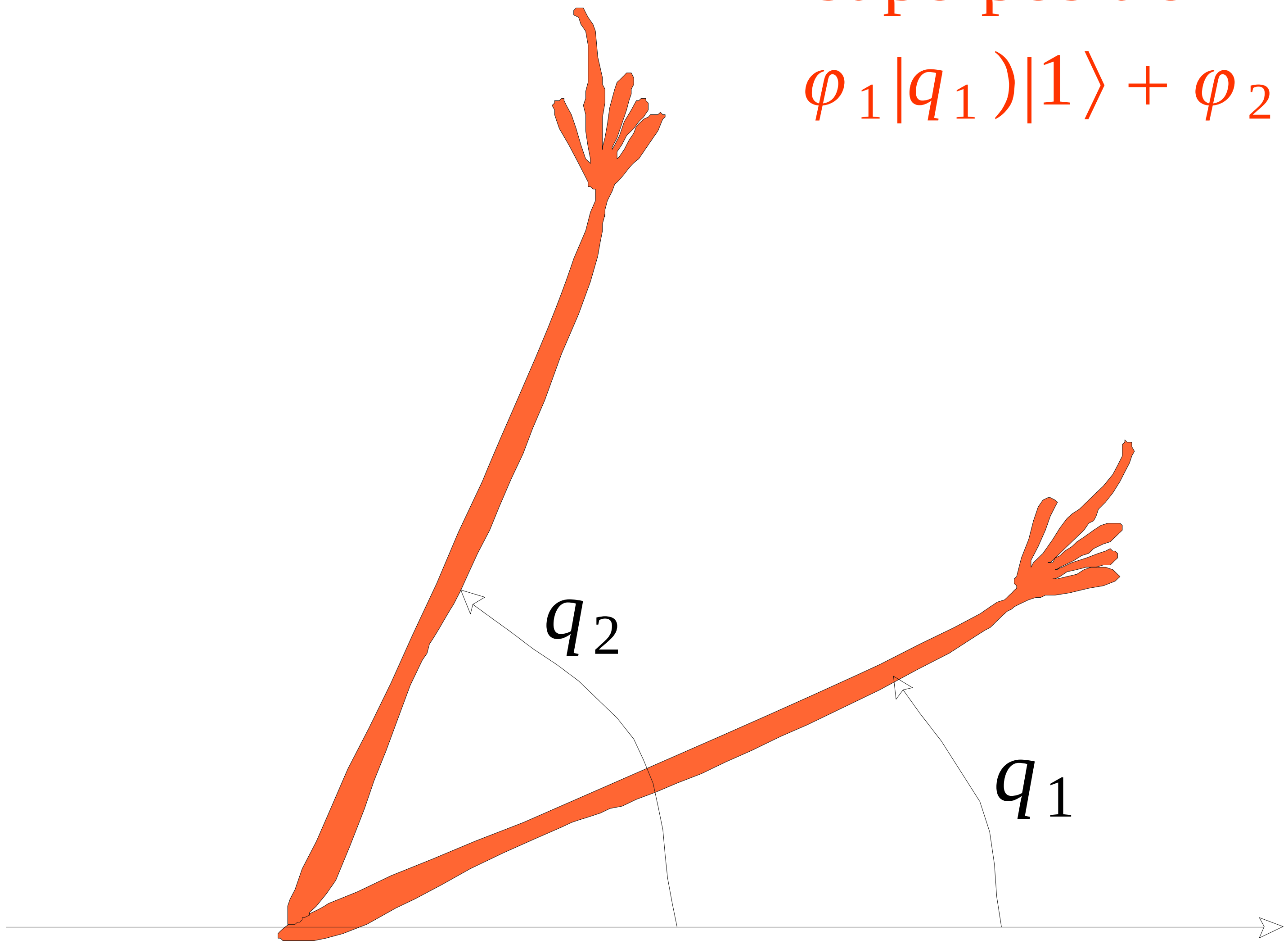
$$\mathrm{e}^{-\mathrm{i}\varepsilon\hat{\xi}\hat{p}\tau/\hbar}|\varphi\rangle \otimes |0)$$
$$= \sum_n \varphi_n|n\rangle \otimes e^{-\mathrm{i}\varepsilon\xi_n\hat{p}\tau/\hbar}|0) \equiv \sum_n \varphi_n|n\rangle \otimes |q_n)$$

$$(0|\, e^{+\mathrm{i}\varepsilon\xi_n\hat{p}\tau/\hbar}\hat{q}e^{-\mathrm{i}\varepsilon\xi_n\hat{p}\tau/\hbar}|0) = (0|\hat{q}\,|0) + \varepsilon\xi_n\tau \equiv q_n$$

$\varepsilon\tau$ must be so large that $|q_n - q_m| \gg \Delta q, \lambda$ and that $|q_n - q_m|$ cannot be blurred by pointer reading

Schrödinger cat type superposition

$$\varphi_1 | q_1) |1\rangle + \varphi_2 | q_2) |2\rangle$$



decoherence

by pointer-bath interaction $H_{PB} = \hat{q}\hat{B}$

bath coupling agent B must contain many additive terms

for oscillator bath, $\hat{B} = \sum_{\mu} \varepsilon_{\mu}\hat{q}_{\mu}$, with $\hat{q}_{\mu}$ coordinate of μ-th oscillator

such interaction decoheres macroscopic superposition to mixture for preliminary discussion, let H_{PB} be switched on only after entanglement and act exclusively; bath uncorrelated with object and pointer initially

$$(q|\,\mathrm{Tr}_B\; e^{-i\hat{q}\hat{B}t/\hbar}\sum_{nm}\varphi_n\varphi_m^*|n\rangle\langle m|\;|q_n)(q_m|\,\rho_B\, e^{i\hat{q}\hat{B}t/\hbar}\;|q')$$

$$= \sum_{nm}\varphi_n\varphi_m^*|n\rangle\langle m|(q|q_n)(q_m|q')\left\langle e^{-i(q-q')\hat{B}t/\hbar}\right\rangle$$

$$\approx \sum_{nm}\varphi_n\varphi_m^*|n\rangle\langle m|\;(q|q_n)(q_m|q')\left\langle e^{-i(q_n-q_m)\hat{B}t/\hbar}\right\rangle$$

decoherence factor $\left\langle e^{-i(q_n - q_m)\hat{B}t/\hbar} \right\rangle$

since B assumed additive in many pieces, central limit theorem yields Gaussian statistics; let $\langle \hat{B} \rangle = 0$

$$\left\langle e^{-i(q_n - q_m)\hat{B}t/\hbar} \right\rangle = e^{-(q_n - q_m)^2 \langle \hat{B}^2 \rangle t^2 / 2\hbar^2} = e^{-(t/\tau_{dec})^2}$$

$$\tau_{dec} = \frac{\hbar\sqrt{2}}{|\vec{q}_n - \vec{q}_m| \sqrt{\langle \hat{B}^2 \rangle}}$$

after exceedingly small time, off-diagonal terms negligible, while diagonal terms remain constant in time

measurement complete

after object-pointer entanglement and decoherence

$$\rho_{OP} \sim \sum_n |\varphi_n|^2 \, |n\rangle\langle n| \otimes |q_n)(q_n|$$

macroscopic mixture: different eigenstates of measured observable uniquely correlated with macroscopically distinct pointer displacements;

no relative coherence left, only probabilities!

generalization

thus far assumed:

$$\boxed{\begin{array}{c} \tau_{ent} \ll \tau_{dec} \ll \tau_{O,P,B} \\ \\ \text{pointer \& bath initially uncorrelated} \end{array}}$$

more realistic $\tau_{ent},\ \tau_{dec} \ll \tau_{O,P,B}$

even better $\tau_{ent},\ \tau_{dec},\ \tau_{B} \ll \tau_{O,P}$

and pointer & bath in mutual equilibrium initially

$$\tau_{ent},\ \tau_{dec} \ll \tau_{O,P,B}$$

concurrence of
entanglement & decoherence

$H_{OP} + H_{PB} = \varepsilon\hat{\xi}\hat{p} + \hat{q}\hat{B}$ no problem:

$$e^{-i(\varepsilon\xi\hat{p}+\hat{q}B)t/\hbar} = e^{-i\xi\hat{p}t/\hbar}\, e^{-i\hat{q}Bt/\hbar}\, e^{-i\varepsilon\hat{\xi}t^2\hat{B}/2\hbar}$$

essentially same discussion, but now mixture of macroscopically distinct states arises directly, without detour through superposition à la Schrödinger cat

$$\tau_{ent},\ \tau_{dec},\ \tau_B \ll \tau_{O,P}$$

concurrence of
entanglement, decoherence & bath correlation decay

$$\mathrm{e}^{-\mathrm{i}(H_B+\varepsilon\xi\hat{p}+\hat{q}B)t/\hbar} \sim \mathrm{e}^{-\mathrm{i}H_B t/\hbar}\,\mathrm{e}^{-\mathrm{i}\xi\hat{p}t/\hbar}\left(\mathrm{e}^{-\mathrm{i}\int_0^t d\tau(\hat{q}+\varepsilon\xi\tau)\hat{B}(\tau)/\hbar}\right)_+$$

if B and H_B both sums of many independent terms,
central limit theorem still applies

$\longrightarrow$ essentially same discussion mutual equilibrium of pointer and bath initially

$$\mathrm{e}^{-\beta(H_B+H_P+\hat{q}\hat{B})} \sim \mathrm{e}^{-\beta H_P/2}\, \mathrm{e}^{-\beta(H_B+\hat{q}\hat{B})}\, \mathrm{e}^{-\beta H_P/2}$$

high-temperature limit, excellent aproximation for macroscopic pointer, relative error $O(\hbar^2\beta^2/\tau_P^2)$

essentially same discussion

final embellishment: drop harmonic oscillator potential for pointer in favor of

$V(q)$ with ``metastable'' dip at $q=0$, finite width and barrier height a little larger than $1/\beta$, and lower flatland outside

then object-pointer interaction only has to get pointer out of dip; amplification of pointer displacements achieved by $V(q)$

f.a.q.

Q: why does single run yield unpredictable single pointer displacement?

A: transition probabilty for $|q_m\rangle \rightarrow |q_n\rangle$

for oscillator model is exponentially small like

$$e^{-|q_n - q_m)|^2/\Delta q^2}, \quad e^{-|q_n - q_m)|^2/\lambda^2},$$

therefore no transitions between different characteristic pointer displacement after decoherence time

conclusion

measurement demystified:

what used to be an axiom for the founders of QM has become a well understood consequence of Schrödinger's equation for compound dynamics

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